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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. DONNELL, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms, of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

February 1948 **Vol. 1, No. 2**

International Congresses—Summer, 1948

Seventh International Congress of Applied Mechanics

The Congress will be held at the Imperial College of Science and Technology, South Kensington, London, England, on September 5 to 11, 1948.

The technical sessions will be organized under the following sections: (1) Elasticity and Plasticity; (2) Aerodynamics, Hydrodynamics, Meteorology; (3) Thermodynamics, Heat Transfer, etc.; (4) Vibrations, Lubrication, and Experimental Methods. In addition, as many as ten general lectures or surveys are contemplated on subjects likely to be of interest to members.

Arrangements have been made for visits to the National Physical Laboratory, the Royal Aircraft Establishment, the National Gas Turbine Establishment, and the General Electric Research Laboratories during the week following the Congress.

Forms of application for membership, and other material concerning the International Congress of Applied Mechanics are now available and can be obtained from the Organizing Secretary, Seventh International Congress of Applied Mechanics, Imperial College of Science and Technology, London, S.W.7.

Second International Conference on Soil Mechanics and Foundation Engineering

The meetings and lectures of this Conference will be held in the Palace Theatre, Zomerhofstraat, Rotterdam, Holland, on June 22 to 26, 1948.

Meetings on earth constructions, earth pressure, ground water, raft and pile foundations will be held June 23; on laboratory and field investigations, June 24; on roads, runways, soil stabilization and theoretical subjects, June 25. Visits to the Delft Laboratory have been arranged for June 22, 23, and excursions from June 26 to 29. It is planned to publish six volumes of Proceedings, involving some 500 papers and priced at 75 guilders per set.

Further information can be obtained by addressing the Conference in care of the Laboratory of Soil Mechanics, Oostplantsoen 25, Delft, Holland.

1948 Congress, International Association for Bridge and Structural Engineering

This Congress will be held in Liége, Belgium, during the first half of September, 1948. Only members of the IABSE may take part. The meetings will be organized under the following subjects: (1) Assembling devices and structural details in steel structures; (2) Developments in building structures in concrete and masonry; (3) Developments in long-span steel bridges; (4) Slabs and various curved structures of reinforced concrete; (5) Analysis of safety and effect of dynamic forces. The contributions will be published in English, French, or German, with summaries in the other two languages.

Further information may be obtained from Prof. Dr. F. Stüssi, Swiss Federal Institute of Technology, Zurich, Switzerland.

General Theoretical and Experimental Methods

(See also Revs. 226, 247, 261, 271, 291, 298, 313, 317, 356)

201. Abraham Many and Saul Meiboom, "An electrical network for determining the eigenvalues and eigenvectors of a real symmetric matrix," *Rev. sci. Instrum.*, Nov. 1947, vol. 18, pp. 831–836.

As the author states, the paper describes an electrical network for determining the eigenvalues and eigenvectors of a real symmetric matrix of order n. It consists of n LC-circuits, each one coupled to each of the others by two condensers. The values of the condensers are adjusted according to the elements of the matrix. Both positive and negative values of the elements can be dealt with by changing the connections of the coupling condensers. The network is excited by a variable frequency source, the resonance frequencies being observed on an oscillograph. The eigenvalues are given by these resonance frequencies, and the components of the eigenvectors by the voltages across the coils.

The construction of the network and the operating procedure are described. The theory of the network when losses in the coils are taken into account is also given, and the case of multiple roots is discussed. The accuracy of the eigenvalues obtained is stated to be better than 0.1 per cent of the greatest eigenvalue, while the accuracy of each eigenvector is about 1 per cent of its greatest component. Herbert Harris, Jr., USA

202. C. Concordia and G. K. Carter, "D-C network-analyzer determination of fluid-flow pattern in a centrifugal impeller," *J. appl. Mech.*, June 1947, vol. 14, pp. 113–118.

The authors consider the problem of determining the flow pattern for the flow of an incompressible ideal fluid through a two-dimensional centrifugal impeller. The first step is to replace the differential equation $\nabla^2\psi = 0$ (ψ being the stream function), expressed in polar co-ordinates, by a corresponding difference equation. By interpreting the latter equation as the summation of currents at a point in a rectangular electrical network, the problem can be set up on a d-c network analyzer, as discussed by H. A. Peterson and C. Concordia ["Analyzers—for use in engineering and scientific problems," *Gen. elect. Rev.*, Sept. 1945, vol. 48, no. 9, pp. 29–37].

Suitable boundary conditions are imposed along the impeller blades and along two concentric circles inside and outside the ring

of blades. These may be applied separately and the results added; thus a variety of cases may be handled at one time. The results obtained for a particular impeller under different conditions of flow are discussed and illustrated by figures.

W. M. Kincaid, USA

203. J. Crank and P. Nicolson, "A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type," *Proc. Camb. phil. Soc.*, Jan. 1947, vol. 43, part 1, pp. 50–67.

The paper deals with methods for obtaining numerical solutions of the partial differential equations:

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} - q \frac{\partial w}{\partial t}$$

$$\frac{\partial w}{\partial t} = -kwe^{-A/\theta}$$

subject to usual boundary conditions in a one-dimensional heat-flow problem. Two known methods are described, involving replacement in one case of the x and in the other of the t derivatives by finite-difference approximations and mechanical integration of the resulting ordinary differential equations in the other independent variable.

A third method is investigated in which both x and t derivatives are replaced by finite-difference approximations. A discussion of the errors thereby introduced is given, with particular reference to their oscillatory amplification when overlapping time intervals are used and to their damping out when nonoverlapping time intervals are used.

D. N. Allen, England

204. W. K. Ergen, "Bridge-type electrical computers," *Rev. sci. Instrum.*, Aug. 1947, vol. 18, pp. 564–567.

It is shown how a Wheatstone bridge may be used to solve quadratic equations. The usual bridge solves the equation $\chi_1 \chi_2 - \chi_3 \chi_4 = 0$ by representing the four variables by the four resistances of the bridge. When the bridge is balanced the equation is satisfied.

More generally, if the resistance of each arm is made equal to an algebraic sum of the form $R_i = \sum_{j=1}^{j=4} a_{ij}\chi_j$, the bridge will solve more complicated quadratic equations. The method is however limited to equations of rank 4 and signature zero.

The method is very useful for analog computers used for gun sights and automatic navigation systems.

Herbert Harris, Jr., USA

205. W. M. Kincaid, "Numerical methods for finding characteristic roots and vectors of matrices," *Quart. appl. Math.*, Oct. 1947, vol. 5, pp. 320–345.

The paper is a condensed version of a thesis submitted at Brown University in 1946. Starting from an earlier article by A. C. Aitken [*Proc. roy. Soc. Edinburgh*, 1937, vol. 47, pp. 269–304] and that of T. C. Fry [*Quart. appl. Math.*, 1945, vol. 3, pp. 89–105] the author treats practical numerical methods for finding the largest characteristic root and corresponding vector, and the other roots and vectors of a matrix.

At first these problems are resolved by the method of iteration for symmetric matrices, and then for general (unsymmetric) matrices having real or complex roots. Different devices for improving convergence and an extension of Aitken's δ^2 process are discussed. If approximate values of the roots are known, the process of iteration can be simplified. The last section of the paper treats the numerical solution of algebraic equations by Bernoulli's matrix method.

All the methods discussed are clarified by concrete numerical examples.

Z. Horak, Czechoslovakia

206. O. L. Bowie, "A least-square application to relaxation methods," *J. appl. Phys.*, Sept. 1947, vol. 18, pp. 830–833.

The author has applied the method of least squares to the relaxation techniques sometimes used in the solution of partial differential equations from highly irregular shapes. A systematic procedure has been devised for weighting errors at various points of the lattice used, wherein the mean-square error is minimized in considering any correction to a given set of values for the unknown function. Application of this procedure to the solution of certain equations will speed up the rate of convergence of relaxation methods over that obtained by use of the usual "point-by-point" procedures.

H. R. Neifert, USA

General Dynamics, Kinematics, Friction

(See also Revs. 245, 305)

207. E. Egervary, "On a generalization of the Lagrangean solution of the problem of three bodies" (in English), *C. R. Acad. Sci. URSS*, Mar. 30, 1947, vol. 55, no. 9, pp. 793–795.

The paper deals with certain particular solutions of the Lagrangean equations referring to the planar three body problem in which the ratio of the principal moments of inertia of the system remains constant.

Introducing the generalized co-ordinates S, D, σ and δ, which are connected to the complex co-ordinates z_k ($k = 0,1,2,$) by the relation

$$\sqrt{3}\, z_k = S\, e^{i\left(\sigma + \frac{2k\pi}{3}\right)} + D\, e^{i\left(\delta - \frac{2k\pi}{3}\right)}$$

the author deals with solutions where $S/D = const = 1$, and proves that the potential function occurring in the author's and Sokolov's solution is (besides the elastic potential) the only one belonging to this category.

Nicholas Sag, Australia

208. H. Poritsky, C. W. Hewlett, Jr., and R. E. Coleman, Jr., "Sliding friction of ball bearings of the pivot type," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 261–268.

The frictional resistance developed in pivot-type ball bearings (ball bearings capable of sustaining end thrust) due to slipping of the balls is analyzed for either an axial or an oblique load and for a single bearing or for two bearings on a shaft. The article demonstrates that as the pivot turns, the balls roll and also slip on one of the contact surfaces (usually the outer) by turning about an axis perpendicular to the contact surface.

The expression for the torque is developed by using the Hertz theory for contact of elastic bodies, and assuming the coefficient of friction to be constant. The resulting moment expression involves an elliptic integral. Curves are included to simplify the calculation of the elliptic integral. Sample problems are solved. There appears to be some question about the answer obtained for the first one.

William B. Stiles, USA

209. Rudolf Klanner, "Wheel suspension of motor vehicles for cross-country operation (Die Radaufhängung des geländegängigen Kraftfahrzeuges)," *Schweiz. Bauztg.*, May 31, 1947, vol. 65, pp. 290–294, June 7, 1947, vol. 65, pp. 299–302.

In the first installment, the manner in which a slowly moving vehicle passes over an obstacle is discussed, the obstacle being of

such nature as to raise either one wheel, or a pair of diagonally opposed wheels. Four different types of wheel suspensions are treated. The analysis assumes that the motion is so slow that statical equilibrium is preserved at each instant. Several of the formulas appearing in Table I seem to this reviewer to be incorrect.

The second part of the article deals with the amount of lateral tilt (deflection in roll) encountered while rounding a curve. The deflection characteristics of the four different types of wheel suspensions under the action of centrifugal force are studied.

The effect of nonlinear springing is also investigated, the nonlinearity being such as to provide a constant natural frequency of the sprung mass regardless of the load carried. Finally, the use of spring compensation is studied, with the object of providing the best combination of performance with regard to lateral tilt and operation over obstacles. Martin Goland, USA

210. E. L. Armi, J. L. Johnson, R. C. Machler, and N. E. Polster, "Application of the sliding-thermocouple method to the determination of temperatures at the interface of a moving bullet and a gun barrel," *J. appl. Phys.*, Jan. 1947, vol. 18, pp. 88–94.

With the objective of determining the instantaneous heat of sliding friction in a gun, a cathode-ray oscillograph was used to measure the electromotive force between a bullet and a short section of rifle barrel as the bullet was driven through the bore by a falling weight, and later by a charge of powder. For the test with the falling weight, a check of temperature was made by burying a standard thermocouple in the bore. Fairly good agreement was obtained between the interface temperatures determined by the sliding thermocouple and by the buried thermocouple. Considerable attention was given to theoretical analysis of moving sources of heat and instantaneous and maximum temperatures under sliding contacts. The temperatures measured ranged up to 1000 C.

This sliding-thermocouple method, with possible refinements for accuracy, should have applications in evaluating designs of brakes, clutches, bearings, and pistons, in addition to its use in ordnance.

No electrical circuits are shown in the article, but they seem to have been of a standard type for use with cathode-ray oscillographs. The work was done by the Leeds and Northrup Company, Philadelphia, under contract with a government agency, and details can probably be obtained from that company.

W. C. Johnson, Jr., USA

Gyroscopics, Governors, Servomechanisms

(See Rev. 211)

Vibrations, Balancing

(See also Revs. 205, 209, 235, 270)

211. R. N. Arnold, "The tuned and damped gyrostatic vibration absorber," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 25, pp. 1–15.

Mechanical vibration absorbers usually consist of one or more of the following elements: masses, elastic springs, frictional or damping elements. This paper indicates that the gyroscope can be another very important element, even though engineers rarely consider it as such.

The absorber studied by the author is rather complex, and consists mainly of a gyrostat fitted with elastic springs and a damping mechanism. An extensive theoretical analysis is made of the response of such an absorber to one-mass and two-mass torsional vibratory systems under both forced and self-induced vibrations. The general results are presented in convenient dimensionless charts. Design methods are derived by which the essential dimensions for any given application may be readily determined.

The measure of the power of this absorber is its "dynamic ratio," which increases with the square of the gyro-rotor speed. By using relatively high gyro-rotor speeds, this absorber can be made considerably more effective than either the Lanchester absorber or the conventional tuned and damped absorber. Satisfactory experimental verification of the theoretical results was obtained for one absorber, over a wide range of forced frequencies and at different values of absorber damping. Other experimental results presented were free vibration decay curves of the main torsional system. These curves can be seen to be greatly affected by the gyro-rotor speed.

The design of a full-scale absorber built to suppress vibration of the table of a heavy planer is given, together with some satisfactory trial results. However, this absorber is a complex mechanism, still in its experimental stage, and practical difficulties are expected. Alexander Yorgiadis, USA

212. H. R. Lawrence, "The dynamics of a swept wing," *J. aero. Sci.*, Nov. 1947, vol. 14, pp. 643–650.

This paper presents a method of calculating the natural frequencies and normal modes of vibration of a vibrating swept wing.

The method of analysis is to regard the swept wing as a rod of variable density and cross section having an elastic axis in the form of a plane curve. By the use of Hamilton's principle, the equations of motion of the curved rod are determined. Only oscillations normal to the plane containing the elastic axis are considered.

A bend in the rod is considered as the limiting case of a circular arc. The theory is applied to the determination of the natural frequencies and normal modes of a free-free swept wing, and a method of calculating the response of a swept free-free wing to a system of applied variable loads is developed.

The conditions under which an airplane with a swept wing may be regarded as a rod of variable cross section are examined.

In the opinion of the reviewer, this paper is a valuable contribution to the field of airplane dynamics.

Louis A. Pipes, USA

213. J. A. Haringx, "Vibration-free mountings with auxiliary mass," *Philips Tech. Rev.*, 1947, vol. 9, no. 1, pp. 16–23; no. 3, pp. 85–90.

The first paper reviews the standard method of reducing the displacement amplitude of a body mounted on a vibrating support by the use of a damped, dynamic vibration absorber. The second paper contains data for the lateral rigidity of helical springs to aid in the design of vibration-isolating mounts.

Albert I. Bellin, USA

214. Bernard E. O'Connor, "The viscous torsional vibration damper," *Soc. auto. Eng. Quart. Trans.*, Jan. 1947, vol. 1, pp. 87–97.

The theory of the untuned viscous damper as applied to rotating systems such as crank shafts is developed. It is shown that the use of silicone fluid as a damping medium produces a highly effective viscous damper.

An illustrative example involving a 6-cyl automotive engine is given, in which the performance of such a damper is evaluated.

The author discusses two practical methods for determining the maximum amplitude, the frequency of this maximum amplitude, and the optimum damping. The first method is entirely

rigorous but somewhat lengthy. The second method on the other hand is based upon certain simplifying assumptions and yields sufficiently accurate results with much less labor.

R. L. Bisplinghoff, USA

215. R. C. Lewis and Karl Unholtz, "Simplified method for design of vibration-isolating supports," *Refrig. Engng.*, April 1947, vol. 53, pp. 291–295.

The paper presents a chart giving relations between various nondimensional parameters encountered in the solution of the equations of motion for an elastically supported rigid body. The chart is in effect a space plot upon which the actual outline of the body may be projected. When this is done it becomes a simple matter to determine the effects of various locations of the resilient units on the magnitude of the significant parameters; in fact, for the selected points the complete dynamic solution can be read directly from the chart.

Examples are given showing the use of the chart in specific design problems. These problems are in general the selection and location of resilient units so that all six of the natural frequencies of the body are well below any exciting frequencies, while at the same time the system has adequate stability. The chart enables one to obtain the best position of the mounting points immediately, without resorting to trial and error. Stiffnesses can then be selected which keep the frequencies below any specified value.

Gerald Pickett, USA

216. P. G. Bordoni, "Asymmetrical vibrations of cones," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 146–155.

The asymmetrical modes of vibrations of conical shells are defined as flexural vibrations with radii as nodal lines. A technique is described for exciting these modes by vibrating the cone along and normal to its axis, and for observing the vibration with a special stroboscope. Measurements on paper cones with vertex angles between 45 and 135 deg showed that the seam in the cone had negligible effect on the frequency and that the frequency was approximately equal to that for a flat circular plate, or cone of vertex angle of 180 deg.

More precise measurements on metal cones, in which the frequency was determined with an electrostatic probe, showed excellent agreement with the theoretical frequency of a flat plate, for modes with more than six nodal radii. The frequency was slightly greater than for a flat plate for fewer nodal radii; an empirical formula is given for the correction in frequency.

W. Ramberg, USA

Wave Motion, Impact, Seismology

217. N. V. Zvolinsky, "Plane waves in an elastic semispace covered with a liquid layer" (in English), *C. R. Acad. Sci. URSS*, Apr. 10, 1947, vol. 56, no. 1, pp. 19–22.

This paper deals with the propagation of plane waves in an elastic semispace covered with a layer of inviscid compressible liquid. The surface of the liquid is assumed parallel to the plane boundary of the elastic medium, while the wave fronts are normal to it. Two cases are considered, depending on the relative magnitude of the velocity of propagation of the plane waves, and of the two specific velocities of propagation of waves in the solid. By the use of the boundary conditions, the velocity potential of the motion in the liquid and the potentials of displacement in the solid are expressed as infinite series involving two arbitrary functions.

The paper is clearly motivated by a geophysical problem, but no applications or numerical results are given. It may be compared with work by V. Gogoladze [*C. R. Acad. Sci. URSS*, 1945, vol. 49, nos. 5 and 7] on reflection and refraction of nonstationary waves. See also S. L. Sobolev [*Trans. Seism. Inst. Acad. Sci. USSR*, 1932, no. 18 (in Russian)] and D. J. Sherman [*Trans. Seism. Inst. Acad. Sci. USSR*, 1945, no. 115 (in Russian)].

A. Robinson, England

218. E. G. Fisher, "Lateral vibration and stress in a beam under shock machine loading," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 5, no. 1, pp. 78–89.

As a means of shedding more light on the problem of shock and shock testing of mechanical equipment, the author has analyzed a typical mechanical system (a hinged beam) both theoretically and experimentally. Operational calculus has been employed to solve for the transient behavior of the beam as a result of sudden lateral displacements of its ends. The displacement or excitation chosen for study (similar to a saw tooth) is exemplified by the motion of Naval vessel hulls adjacent to near-by underwater explosion. Reasonable agreement is found between the theory and the actual behavior of a hinged beam attached to a Naval shock-testing machine and studied by wire strain-gage technique.

The author points out the danger in employing peak acceleration, which is only one phase of the problem, as a criterion of shock severity in general.

Applications of such work can undoubtedly be made in fields other than that of equipment attached to Naval hulls. An example would be the study of structural or machine elements of vehicles subject to road shocks.

Charles W. Gadd, USA

219. J. G. Scholte, "On Rayleigh waves in visco-elastic media," *Physica, Hague*, May 1947, vol. 13, pp. 245–250.

A visco-elastic medium is defined, following Larmor and Sezawa, as one for which the elastic constants of Lamé are replaced by the operators $\lambda_1 + \lambda_2 \partial/\partial t$, $\mu_1 + \mu_2 \partial/\partial t$. The plane wave systems that can exist in an infinite medium are set forth, body forces being neglected and, as in the elastic case, are found to be either longitudinal or transverse.

Wave motion in a semi-infinite body ($z \geqslant 0$) is studied and is found to be determined by a cubic equation whose complex coefficients are functions of the Lamé elastic constants λ_1,μ_1, and the corresponding visco-elastic constants λ_2,μ_2; when viscosity is neglected this equation reduces to the well-known cubic for Rayleigh waves in an elastic medium. The author discusses the influence of viscosity on the periodicity of the possible wave systems and their attenuation, both in depth and on the surface. The definition of Rayleigh waves proposed by Hardtwig [*Z. für Geophysik*, 1943, vol. 18, p. 1] is held to be unnecessarily restricted.

Robert D. Specht, USA

220. M. P. White and LeVan Griffis, "The permanent strain in a uniform bar due to longitudinal impact," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 337–341.

A method is presented for predicting the final strained state of a long uniform bar or wire of ductile material, subjected to a longitudinal impact of finite duration during which the impact stress is constant and exceeds the yield strength. The velocity of the wave front as a function of the local stress or strain is calculated from the slope of the stress-strain curve. Similarly that of the wave rear is calculated by assuming that Hooke's law with the same modulus of elasticity as for small strains applies during unloading. The particle velocity may then be obtained, integrating the wave velocity over the strain. This procedure may be extended also to the unloading process, when the back of the wave passes the point in question.

It is thus shown that the final strains are greatest immediately adjacent to the impact point, and are constant over a finite length

of specimen, which corresponds to the distance over which the back of the wave overtakes the front. Following this region of constant residual strain is a region, also of finite length, in which the permanent strain decreases with distance from the end of the specimen. At the end of the region of decreasing stress, there is, due to the action of a secondary wave which is reflected from the original point of impact, another zone of constant permanent strain, which is followed by one of decreasing permanent strain, and so on.

It is demonstrated that in tensile impact a critical velocity of impact can be defined, corresponding to the ultimate strength of the material. At impact velocities greater than this critical velocity, rupture occurs quickly and close to the point of impact, so that the remainder of the specimen does not receive large deformations. Thus the energy required to break a specimen in tension is less at supercritical than subcritical velocities.

Folke K. G. Odqvist, Sweden

Acoustics

(See also Revs. 216, 293)

221. W. P. Mason and H. J. McSkimin, "Attenuation and scattering of high-frequency sound waves in metals and glasses," *J. acoust. Soc. Amer.*, May 1947, vol. 19, pp. 464–473.

In this paper a comparison is made between the experimental and theoretically calculated velocity and attenuation of longitudinal and shear waves in rods of aluminum and glass. High-frequency energy was introduced into the rods in pulses by an appropriately vibrating quartz crystal waxed to one end of the rod, while the decaying reflected pulses were picked up by a similar crystal affixed to the other end.

The measured attenuation contained terms proportional to the first and fourth powers of frequency, corresponding to elastic hysteresis and scattering losses. The latter were calculated from the Rayleigh relation, the space-averaged elastic constants, and the particle size of the material. In aluminum rods the calculated shear waves agreed especially well with the measured results. The fourth-power law was found valid for particle sizes below one-third wave length. In glass rods the fourth-power term was absent, indicating very small effective particle sizes in the frequency range up to 20 megacycles per sec.

Vincent Salmon, USA

222. R. H. Bolt, "Normal frequency spacing statistics," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 79–90.

The author continues his series of contributions to the theory of the new room acoustics. The theory of normal frequency distribution in rooms as thus far presented has been concerned mainly with smooth *average* expressions. However, when the room is small in terms of wave length, only a few normal modes are excited, and fluctuations (characterized by transmission irregularity) are marked.

This paper extends the theory of statistical methods to include the fluctuation properties. The study is generalized by dimensionless treatment to yield results independent of volume but explicitly dependent on shape. A "frequency spacing index" is defined and evaluated for random and uniform distribution and for cubical and rectangular rooms whose dimensional ratios can be handled by number theory. The index is a direct measure of the irregularity of response, and an inverse measure of the degree of diffusion or approach to randomness.

Formulas are also given so that the index can be calculated for other cases if the normal frequency values are available. It is shown that the index always tends to increase with frequency for smooth rectangular rooms. This is in accordance with a previous conclusion of Morse and the author to the effect that sound never becomes thoroughly diffuse in a rectangular room, but actually becomes less diffuse as frequency is increased. It must be concluded that random distribution of the normal frequencies will be attained only if *sufficient* perturbation is present in boundary shape or value. (This reviewer notes that means currently employed to introduce perturbation include nonparallel walls, nonplanar surfaces, and irregularly distributed areas of sound-absorbing material of varying size and coefficient.)

The study indicates that a relatively broad range of room proportions may be as satisfactory as the currently accepted cube root of 2 and 2:3:5 ratios of dimensions. The quantitative effects on response, irregularity of damping, and other factors neglected in this study remain to be investigated. R. P. Glover, USA

223. L. W. Labaw and A. O. Williams, Jr., "Absorption of supersonic waves in water near one megacycle," *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 30–34.

The authors tentatively conclude (from experiments admittedly characterized by large experimental error) that it is improbable that there is an absorption peak in water in the region of one megacycle, as suggested by other workers. Disparities between the intensity absorption coefficients predicted from the classical theory of Stokes and those obtained experimentally persist in the frequency region from 1 to 58 megacycles at least; measured values are of the order of two to three times the theoretical values. R. P. Glover, USA

224. R. H. Bolt, "On the design of perforated facings for acoustic materials," *J. acoust. Soc. Amer.*, Sept. 1947, vol. 19, pp. 917–921.

This paper brings to the design level the *results* of previous acoustic research so that one of the elusive variables, the facing, may be rationally evaluated. It is a continuation of the work of P. M. Morse and the author [*Rev. mod. Phys.*, 1944, vol. 16, p. 69] which adds to the knowledge of design and choice of room soundproofing. It depends on its references for understanding of terminology. W. E. Trumpler, Jr., USA

225. L. L. Beranek and H. W. Rudmose, "Sound control in airplanes," *J. acoust. Soc. Amer.*, Mar. 1947, vol. 19, pp. 357–364.

This paper is a condensation of several more detailed papers on the subject of noise control in aircraft. An excellent list of references covering the particular field of noise measurement and control in aircraft is included. The following list of section headings indicates the scope of the material covered: (1) Measurement of sound levels in flight; (2) Specification of acceptable noise levels; (3) Sources of noise in airplanes; (4) Variation of noise levels with flight conditions. C. E. Warren, USA

Elasticity Theory

(See also Revs. 208, 243, 246, 255, 268)

226. S. Bergman, "Punch-card machine methods applied to the solution of the torsion problem," *Quart. appl. Math.*, Apr. 1947, vol. 5, pp. 69–81.

Torsional displacements and stresses of an elastic bar are considered. The author applies his "method of orthogonal functions" ["Sur les fonctions orthogonales," Interscience Publishers, New York, 1941] to the approximate determination of the harmonic stress function for a square cross section with cutouts at the corners. The computations were carried out on punch-card ma-

chines; certain techniques, described by R. Lorant in the publications of the International Business Machines Corp., are discussed briefly. A device for obtaining a second approximation is explained. W. M. Kincaid, USA

227. R. B. Heywood, "The relationship between fatigue and stress concentration," *Aircr. Engng.*, Mar. 1947, vol. 19, pp. 81–84.

It is well known that the fatigue-stress-concentration factor k, relating the fatigue strength σ_{fn} of a notched specimen to the fatigue strength σ_f of the unnotched material by means of the formula $\sigma_{fn} = \sigma_f/k_f$, is smaller than the stress-concentration factor k as given by the theory of elasticity for a homogeneous material. This discrepancy is usually attributed to the high stress gradients occurring in the neighborhood of the notch.

The author derives an empirical formula for the fatigue-stress-concentration factor of the form

$$K_f = \frac{K}{1 + \dfrac{a}{\sqrt{br}}}$$

where a is a material constant, b depends only on the type of notch, r is a dimension which determines the absolute size of the notch. This formula is confirmed to a satisfactory degree of approximation by a comparison with a number of tests.

W. T. Koiter, Holland

228. C. W. MacGregor and L. F. Coffin, Jr., "Approximate solutions for symmetrically loaded thick-walled cylinders," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 301–311.

The exact solutions for the problem of bands of internal or external loading symmetrically applied to a thick-walled cylinder are not in closed form, and the numerical evaluation of the stresses is quite lengthy and tedious.

The objects of this paper are (*a*) to present a simple and practical solution to the stated problems, approximate in nature and yet sufficiently accurate for design purposes; (*b*) to develop this solution in closed form so that a designer can evaluate stresses in a relatively short time; and (*c*) to compare the results of this approximate method with those determined by the exact solutions and by experiments.

The characteristic feature of this solution is that the cylinder is considered as being composed of a number of sectors or "staves," which are treated as bars on an elastic foundation.

R. G. Boiten, Holland

229. J. M. Klitchieff, "Torsion of a rectangular tube," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 287–288.

The well-known solution for the torsion of a solid rectangular tube is applied to discuss the torsion of a hollow square tube and the shape of certain possible internal boundaries is calculated. For the case when the tube is relatively thin, comparison is made between the values of the shear stress and the twist and the values of these quantities predicted by the Bredt-Batho formulas.

H. G. Hopkins, England

230. Chih-Bing Ling, "Stresses in a notched strip under tension," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 275–280.

The stress analysis in a strip notched by circles has long attracted the attention of the elastician as well as the engineer. A good deal of experimental work has been done by various investigators, especially by the photoelastic method.

Although some approximate methods previously may have been used to calculate the stresses in a notched strip with or without the aid of photoelastic observations, no previous theoretical solution seems to have been developed. The author has now formulated such a solution for a strip under tension, the strip being notched by a pair of symmetrical semicircular arcs on its boundaries. The solution is very ingenious and involves a stress function constructed with functions derived from two appropriate solutions of an unnotched strip. Numerical examples of the solution have been worked out and compared with available experimental results. Emilio Pittarelli, USA

231. S. G. Gutman, "General solution of the problem of the theory of elasticity in generalized cylindrical co-ordinates" (in Russian), *Notes Acad. Sci. USSR* (*Doklady, Ak. Nauk, SSSR*), Nov. 21, 1947, vol. 58, no. 6, pp. 993–996.

Generalized cylindrical co-ordinates are orthogonal co-ordinates, α, β, γ, which have the differential parameters $h_\alpha = h_\beta = h = \sqrt{h_\gamma}\,h_1$ and $h_\gamma = (a\gamma + b)^2$ where $\partial h_1/\partial\gamma = 0$. This includes two systems of co-ordinates: (1) a cylindrical system α, β, z for $a = 0$, $b = 1$, $\gamma = z$, $h_\gamma = 1$, and (2) a spherical system α, β, $-1/R$ for $a = 1$, $b = 0$, $\gamma = -1/R$, $\alpha = \ln tg(\varphi/2)$, $\beta = \lambda$ (length), $h_\gamma = \gamma^2 = 1/R^2$, $h = 1/(R \sin\varphi)$.

The components of rotation ω are determined by the equations

$$2G\frac{\omega_\alpha}{h} = \frac{\partial}{\partial\alpha}\left[h_\gamma \frac{\partial}{\partial\gamma}\left(\frac{F_\omega}{\sqrt{h_\gamma}}\right)\right] - (1-\sigma)\frac{\partial}{\partial\beta}\left(\frac{\Delta F_s}{\sqrt{h_\gamma}}\right)$$

$$2G\frac{\omega_\beta}{h} = \frac{\partial}{\partial\beta}\left[h_\gamma \frac{\partial}{\partial\gamma}\left(\frac{F_\omega}{\sqrt{h_\gamma}}\right)\right] + (1-\sigma)\frac{\partial}{\partial\alpha}\left(\frac{\Delta F_s}{\sqrt{h_\gamma}}\right)$$

$$2G\frac{\omega_\gamma}{h} = \frac{\partial}{\partial\gamma}\left[h_\gamma \frac{\partial}{\partial\gamma}\left(\frac{F_\omega}{\sqrt{h_\gamma}}\right)\right.$$

where G is the modulus of rigidity, σ Poisson's ratio, F_ω a harmonic function ($\Delta F_\omega = 0$), F_s a biharmonic function ($\Delta^2 F_s = 0$), Δ the Laplace operator.

For the components of the displacement u we have

$$2Gu_\alpha = h\left[\frac{\partial\Phi}{\partial\alpha} + \frac{\partial}{\partial\beta}\left(\frac{2F_\omega}{\sqrt{h_\gamma}}\right)\right]$$

$$2Gu_\beta = h\left[\frac{\partial\Phi}{\partial\beta} - \frac{\partial}{\partial\alpha}\left(\frac{2F_\omega}{\sqrt{h_\gamma}}\right)\right]$$

$$2Gu_\gamma = h_\gamma\frac{\partial\Phi}{\partial\gamma} - 2(1-\sigma)\frac{\Delta F_s}{\sqrt{h_\gamma}}$$

where $\Phi = \sqrt{h_\gamma}\,\dfrac{\partial F_s}{\partial\gamma} - (3-4\sigma)\dfrac{\partial\sqrt{h_\gamma}}{\partial\gamma}F_s$ is a biharmonic function ($\Delta^2\Phi = 0$). Dr. Z. Bazant, Czechoslovakia

232. K. H. Swainger, "Large strains and displacements in stress-strain problems," *Nature, Lond.*, Sept. 20, 1947, vol. 160, pp. 399–400.

In this short note (a treatise on the complete general treatment is promised) the author defines "true" strain as the ratio of change in length to the deformed length and "true" stress as the ratio of force to actual area, defines Poisson's ratio and Young's modulus in terms of these, and then gives a general relation between stresses and finite displacements. The theory may be of use in describing the large strains frequently measured in structural tests. W. L. Esmeijer, Holland

233. G. S. Shapiro, "Stress functions in an arbitrary system of curvilinear co-ordinates (Les fonctions des tensions dans un système arbitraire de coordonnées curvilignes)," *C. R. Acad. Sci. URSS*, Mar. 20, 1947, vol. 55, pp. 693–695.

The relations between the Galerkin vector [*C. R. Acad. Sci. URSS, A*, 1930, vol. 14, p. 353] and the functions introduced

by H. Neuber [*Kerbspannungslehre*, 1937, p. 19] are studied. The transformations of displacement and stress in the theory of the Galerkin vector are given, using arbitrary curvilinear coordinates. Folke K. G. Odqvist, Sweden

Experimental Stress Analysis

(See also Revs. 271, 364)

234. Carl L. Frederick, "Aircraft instruments for radio-telemetering and television-telemetering," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 4, no. 2, pp. 103–121.

Instruments for telemetering data from an airplane in flight to the ground are described in considerable detail. Emphasis is on the design of the pickups, rather than on the electrical systems for telemetering the data. Two transmission systems were used, frequency modulated radio waves for data which varied rapidly and television for data which varied slowly.

The instruments described include those used to measure structural strains, aerodynamic pressures, linear and angular accelerations, rates of pitch and roll, altitude, airspeed, dive angle, angle of yaw, temperature, voltage, current, and position of control surfaces, and to indicate breakage. Those described in most detail include the stick force indicator, rate of roll indicator, and yawmeter. The basic pickup element used in most of the instruments was the wire resistance strain gage.

An experimental method of studying flutter is described briefly, and sample records are shown. The paper is well illustrated.

B. F. Langer, USA

235. S. S. Manson, R. H. Kemp, and W. C. Morgan, "High-temperature strain gages and their application to measurement of vibratory stresses in turbosupercharger buckets," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 5, no. 1, pp. 90–100.

The conventional bakelite-impregnated wire strain gage is usually limited to temperatures below 500 F. For temperatures up to 1500 F the authors found that nichrome V wire is suitable because of its high specific resistance, stable strain-sensitivity factor, and resistance to breakdown and corrosion at elevated temperatures. Sauereisen No. 1 cement proved adequate to 1500 F. Woven glass tape was used for forms and was found satisfactory only to 1200 F. Techniques are described for forming gages with a gage length as low as $^1/_{32}$ in.

The gages tested appeared to have stable characteristics after the first cycle of heating and cooling. Polarity of the strain gage was found to be very important. For connections silver-graphite brushes operating on monel-metal slip rings were found satisfactory, but all four gages forming the Wheatstone bridge should be mounted on the rotating part to minimize slip-ring effects. A description is given of the use of the strain gages to study vibration in turbine buckets of turbosuperchargers operating at speeds up to 20,000 rpm. Glenn Murphy, USA

236. R. F. Hanstock and E. H. Lloyd, "An x-ray method of measuring Poisson's ratio," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 26, pp. 52–54.

The authors apply the x-ray diffraction method to a simple tension specimen, and measure, under various stresses, the spacing between parallel crystal planes oriented at a succession of angles to the tension. By plotting the "rate of change in spacing with stress" against the angle, they determine the direction of zero strain. In an isotropic material this occurs at an angle ψ to the normal to the tension such that $\cos 2\psi = (1\sigma)(-1+\sigma)$ where σ = Poisson's ratio. At lesser angles shortening occurs, and at greater angles elongation. For a heat-treated aluminum alloy the values of Poisson's ratio so obtained ranged from 0.343 to 0.379, compared to a mechanically determined value of 0.35.

Discussers point out that since crystal grains are anisotropic x-ray methods could not always be expected to check mechanical methods. The reviewer adds that aluminum seems more nearly isotropic than most metals. Experimental results with the more anisotropic metals might be desirable, as a step in a general study of stress distribution from grain to grain. The reviewer was mainly impressed by the terse description of the x-ray method and its promise of possible high accuracy in the determination of trapped stresses, rather than of Poisson's ratio.

Mortimer F. Sayre, USA

237. J. T. Norton and D. Rosenthal, "Recent contributions to the x-ray method in the field of stress analysis," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 5, no. 1, pp. 71–77.

The authors discuss the possibilities of the x-ray method of stress determination in the field of experimental stress analysis, emphasizing its limitations as well as its advantages. Specific applications and techniques of this method are reviewed from the authors' numerous papers published in the *Welding Journal* since 1944. F. J. Mehringer, USA

238. A. Boodberg and E. D. Howe, "Method of obtaining the stress at the mid-thickness by measurements from only one surface of a plate," *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 5, no. 1, pp. 56–58.

In lieu of placing strain gages on the top and bottom surfaces of a plate and averaging the results, gages were placed on the top surface and at a known distance above, preferably half the plate thickness.

The upper SR4 gage was supported on a thin steel "bridge strip," spot-welded near its ends to clearance blocks. These were held by stud bolts, welded to the plate. The desired initial tension in this strip could be obtained by adjusting a lateral screw in one of the clearance blocks. A second bridge strip was placed at right angles to the first one, and an SR4 strain rosette was placed on the surface of the plate below the bridge strips. Knowing the strain at the surface and the bridge-strip elongation at a known distance above, the strain (or stress) at mid-point of the plate could be computed.

Laboratory test results listed in the article showed no more than 3 per cent deviation between center line stresses obtained by this method and those obtained by averaging SR4 gage readings taken on opposing faces of the plate. M. F. Sayre, USA

239. C. R. Urwin and K. H. Swainger, "Minimizing zero drift in electrical strain-gage bridges," *J. roy. aero. Soc.*, Nov. 1947, vol. 51, pp. 867–873.

The factors which produce zero drift in wire strain-gage circuits are discussed, and bridge circuits are suggested which minimize this type of error. The use of high-resistance ratio arms allows coarse resistance changes at the measuring bridge to measure the small resistance change at the gage due to strain. High potentials are applied to the bridge to minimize the effects of thermal emf. B. F. Langer, USA

240. M. H. Polzin, "Unveil plane wheel service stresses with laboratory stresscoat analysis," *Soc. auto. Eng. J.*, June 1947, vol. 55, pp. 26–28, 32.

The paper describes qualitative tests of stress distribution in spokes of airplane wheels by use of brittle coatings. In the most highly strained regions indication is obtained of the direction of principal stresses, which may be measured quantitatively by

subsequent application of strain gages. An attempt is also made to analyze strains due to loads changing in value and direction, by coatings of different sensitivities and by superimposing patterns.

M. Hetényi, USA

241. Antonio de Sousa Continho, "Determination of stresses in concrete with a photoelastic gage (Determinacaó de tensões no betão pelo método do tensómetro fotoelástico)," *Inst. Superior Tecn. Lisbon*, 1947, publ. no. 5, pp. 1–39.

Mabboux suggested in 1935 the use of small glass mirror-disks embedded in the surface of concrete structures for determining the stresses. Tesar suggested making a hole in the disk and using a portable polariscope to determine the principal stresses at the point. The author has carried out these suggestions by designing a polariscope and making calibrations with concrete blocks in compression tests. The author was mainly interested in determining the stresses produced by external loads, although mechanical, acoustical, and optical gages seem better suited for this purpose. On the other hand the photoelastic gage is the only one able to measure stresses produced by shrinkage, creep, and the weight of the structure.

Three applications are described to membranes and tunnels. The calculated and measured values are very different, although the author claims an accuracy of 15 per cent for the photoelastic gage.

A. J. Durelli, USA

242. C. B. Norris and A. W. Voss, "An improved photoelastic method for determining plane stresses," *Nat. Advis. Comm. for Aero. Tech. Note No. 1410*, Jan. 1948, pp. 1–43.

Drucker's method of oblique incidence allows the separation of principal stresses in photoelastic analysis. In this paper the authors expand Drucker's work, suggesting the taking of an additional oblique photograph and the use of simpler equations than Drucker's, with the object not only of increasing the accuracy of the determination of the principal stresses, but of calculating their direction as well.

A description of a polariscope, suspended from tracks, and able to rotate in two perpendicular directions is given. The new method is applied to the study of the stress distribution in a disk under diametral compression and the values obtained check well with those obtained by theory. For the determination of the direction of the principal stresses however the authors found the brittle-coating method easier and more accurate.

A. J. Durelli, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 209, 213, 218, 226, 229, 261, 267, 268, 282)

243. A. K. Rukdadze, "Deformation of a naturally twisted rod" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Sept.–Oct. 1947, vol. 11, pp. 533–542.

The problem of the equilibrium of long naturally twisted rods of an arbitrary cross section was considered by P. M. Riz [*Bull. Acad. Sci. URSS, Ser. Math.*, 1939, no. 4] and by A. Lurje and G. Janelidze [*C. R. Acad. Sci. URSS*, 1939, vol. 24, nos. 1, 3, 4; 1940, vol. 27, no. 5]. Riz considered the problems of extension, torsion, and bending by couples by a method of perturbations of a small parameter characterizing the uniform twist. Lurje and Janelidze used the same method to solve the problem of bending by a transverse force, and found that their results appeared in a form inconvenient for computations.

In a previous paper the author [*Appl. Math. Mech.*, 1942, vol. 6] proposed a method of solution of these problems using the theory of functions of a complex variable. He considered the case of uniform twist determined along the axis of the rod by the function $\theta = kz$ with parameter k so small that one can neglect its second and higher powers. The present paper contains an explicit solution of the problem of bending by a transverse force of naturally twisted rods of elliptical cross section, by the method developed in the author's earlier paper.

I. S. Sokolnikoff, USA

244. Lothar Possner, "The mathematical treatment of the influence lines for simple loads (Die Einflusslinien der einfachen Belastungsfälle und ihre mathematische Behandlung)," *Bauplan. u. Bautech.*, Oct. 1947, vol. 1, pp. 111–116; and Dec. 1947, vol. 1, pp. 193–196.

The deflection of a straight uniform beam under transverse loads satisfies a differential equation of the fourth order. Each section of the beam bounded by points of concentrated loading requires a separate solution of this equation, involving four arbitrary constants, the determination of which is normally quite laborious. The author presents a systematic and compact method for the determination of these constants. Combinations of axial and transverse loadings are also considered.

G. E. Hay, USA

245. E. Czitary, "On the bending stress of cables (Zur biegungsbeamspruchung der drahtseile)," *Ost. Ingen.-Arch.*, 1947, vol. 1, nos. 4 and 5, pp. 342–350.

A kinematic method is used to derive a formula for the bending stress at any section of a wire spirally wound around a central core bent to a given radius. The paper is of interest because the method, while not new, is not well known.

A point is assumed to travel along the wire with a velocity compounded from a simple rotary motion and a motion along the bend. The equation expressing the acceleration required to maintain such a motion is solved for the corresponding radius of curvature, expressed as a function of coil diameter, radius of bend, ratio of component velocities, and distance of the section in question from the approximate neutral surface. The bending stress is obtained by substituting this expression in the conventional beam formula.

In most practical cases the bending stress differs only slightly from that obtained by assuming the radius of curvature of the wire to be the same as the radius of bend. The results can be extended to more complicated cables consisting of spirally wound strands each one of which is itself a bundle of spirally wound wires.

C. W. Smith, USA

246. H. W. F. C. Héman, "Calculation of reinforced concrete tubes with circular cross section under internal pressure with consideration of tensile stresses in the concrete (Berekening van gewapend beton buizen met ronde doorsnede, voor inwendige drukbelasting, met inachtneming der betontrekspanningen)," *Ingenieur, 's Grav.*, Dec. 5, 1947, vol. 59, pp. Bt89–94.

The author derives Lamé's equations for the thick-walled homogeneous tube and applies them to the reinforced concrete tube, conceived as an assembly of an inner and outer concrete cylinder with the reinforcement, represented by a steel cylinder, between them. For mathematical purposes the latter is considered infinitely thin. The tangential stresses (tensions) as well as the radial stresses (compressions) at points of the inner and outer concrete are thereby obtained as functions of the radius of the steel reinforcement.

The greatest tangential stress occurs at points of the inner surface of the reinforced tube, and the nearer the reinforcement is placed to the inner surface the more effective it becomes. Ex-

pressions are derived for the best location, on the basis of the strength of the concrete layer between reinforcement and inner surface. Several examples are given. F. Hymans, USA

247. **E. C. Colin, Jr., and N. M. Newmark, "A numerical solution for the torsion of hollow sections,"** *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 313–315.

The problem consists in the numerical computation of Prandtl's stress function ϕ for the case of a hollow section. In accordance with Liebmann's iteration or Southwell's relaxation method the values of ϕ are determined at the nodes of a network.

Using the well-known contour integral $\int_c \frac{\partial \phi}{\partial n}\, ds$ round the inner boundary, a relation is derived between the value of ϕ at the boundary netpoints and the value of ϕ at the surrounding netpoints. This relation fits in directly with the numerical iteration process of determining all values of ϕ.

W. L. Esmeijer, Holland

248. **Alb. Schlag and André Jorissen, "Elastic behavior of pipes subjected to thermal strains: two-dimensional case (Aptitude élastique des tuyauteries àbsorber les dilatations dues a la température: cas des tuyauteries situées dans un plan),"** *Rev. univ. Min.*, 1947, vol. 3, no. 1, pp. 10–20.

The authors calculate the flexure of pipe lines due to temperature dilatations. Special attention is given to the stress distribution in the curved parts. The article is of special interest for pipeline designers, as they will find a good compilation in it.

J. J. Koch, Holland

Plates, Disks, Shells, Membranes

(See also Revs. 228, 238, 258, 262, 263)

249. **C. M. McDowell, "Stresses in rotating disks by radius of curvature integration,"** *Soc. auto. Eng. Quart. Trans.*, July 1947, vol. 1, pp. 470–475.

The author presents a trial-and-error method of determining the stresses due to centrifugal forces on a rotating disk of variable cross section. A stress function is plotted against the radius, starting at the outside of the wheel. The slope of the curve is assumed at this point, and the curve is constructed by calculating the radius of curvature at convenient intervals along the radius. If the curve so constructed does not satisfy the boundary conditions at the inner radius, a new slope is assumed at the outside of the wheel and the process repeated.

The author finds that the average problem can be solved in three or four trials. The radial and tangential stresses along the radius of the wheel can readily be computed once the curve of the stress function has been constructed. The results obtained for a wheel of hyperbolic outline are compared with an exact mathematical analysis and found to check within five per cent.

Gordon L. Jeppesen, USA

250. **A. M. Binnie, "Stresses in streamline shells due to unsymmetrical loading,"** *Aircr. Engng.*, Apr. 1947, vol. 19, pp. 125–126.

This paper is an extention of Timoshenko's treatment [Theory of Plates and Shells, McGraw-Hill Book Co., Inc., 1940, p. 373] of the axially symmetrical shell subjected to unsymmetrical loads. A solution is presented for cases when the tangential, meridional and normal components of the load on the shell can be represented in the form: $X = A(\phi)d^2F(\theta)/d\theta^2$, $Y = B(\phi)dF(\theta)/d\theta$, $Z = C(\phi)dF(\theta)/d\theta$, respectively. ($\phi$ is the angle in the meridian plane between the normal to the surface and the axis of symmetry, and θ is the angle of longitude about the axis of symmetry.)

For such loads the equations of equilibrium may be reduced to two equations by assuming the meridional tension $N\phi$ and shear force $N_{\theta\phi}$ in the form $N_\phi = P(\phi)\ F(\theta)/d\theta$, $N_{\theta\phi} = Q(\phi)\ F(\theta)$. The resulting two equations may be solved for the functions $P(\phi)$ and $Q(\phi)$. When the integration of these two equations is difficult, P and Q may be found by considering the equilibrium of finite portions of the shell.

The author applies the analysis to a specific shell of revolution, loaded by gravity forces acting perpendicular to the axis of rotation. Paul F. Chenea, USA

251. **M. Dantu, "Note on the calculation of thin shells of revolution with variable thickness (Note sur le calcul des voiles minces de révolution à épaisseur variable),"** *Ann. Ponts Chauss.*, Mar.–Apr. 1947, vol. 117, pp. 241–252.

The differential equations of equilibrium of a thin shell of revolution with variable thickness, loaded by its own weight and by vertical loads symmetrical about the axis of revolution, are integrated: (1) when the laws of variation of the principal stresses (along the meridian and along the parallel) are given, to obtain the meridian curve and the law of variation of the thickness; (2) when the meridian curve and the law of variation of one of the principal stresses (or a relationship between the two principal stresses) are given, to obtain the law of variation of the thickness.

As an example of the first case, the author obtains the meridian curve and the thickness of a shell with constant equal principal stresses and gives limitations to the size of reinforced concrete shells of this type on the basis of both concrete strength and instability. As examples of the second case, the thickness of spherical shell is determined when the meridional stress is constant and when the principal stresses are everywhere equal (shearless shell). M. G. Salvadori, USA

252. **Ezra G. Odley, "Deflections and moments of a rectangular plate clamped on all edges and under hydrostatic pressure,"** *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 289–299.

Two methods for solving the problem of a clamped plate with hydrostatic pressure are compared. In the first method, due to S. Timoshenko, a system of linear algebraic equations is solved to determine coefficients of several Fourier series that define the solution. The second method, due to H. Marcus, is an iteration procedure. Marcus' method is used by the author to compute deflection formulas and tables of bending moments. Tables are given for side ratios of the plate ranging from 0.6 to 2 at intervals of 0.2. For a square plate a comparison is given for bending moments determined by the two methods. A difference of 16 per cent occurs in the maximum negative moment.

S. U. Benscoter, USA

253. **V. Z. Vlasov, "Some new problems of structural mechanics of shells and thin-wall constructions"** (in Russian), *Bull. Acad. Sci. URSS (Ser. techn. Sci.)*, 1947, no. 1, pp. 27–51.

This paper deals principally with cylindrical shells, which are subjected to an arbitrary loading, and are considered as membranes whose sections parallel to the generators have some bending rigidity.

The differential equations corresponding to the principal problems involving the strength and the stability of such cylindrical shells are given, together with general considerations regarding their solution. Open cylindrical shells and prismatical shells are

examined in some detail. The differential equations of equilibrium and deformation for shells of arbitrary shape are also given.

Due to the great range of the subject matter, the author deals rather briefly with each problem he examines.

Leon Beskin, USA

254. A. L. Goldenweiser, "Approximate calculation of thin shells of zero Gauss curvature" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, July–Aug. 1947, vol. 11, pp. 409–422.

The main object of this paper is a qualitative analysis of stressed states in thin elastic shells with developable middle surfaces. The paper also contains an outline of the methods of approximate calculation of stresses. The shell is covered by a net α, β of lines of curvature (the α-lines being of zero curvature) so that the first fundamental form for the surface is of the type $ds^2 = d\alpha^2 + B^2 d\beta^2$. In this case Love's general equations of the shell theory are reducible to two differential equations for the stress functions t and m, from which the forces, moments and deformations can be computed by differentiation.

These equations are

$$\frac{\lambda^2}{B^2}\frac{\partial}{\partial\alpha}B^2\frac{\partial t}{\partial\alpha} - \frac{\lambda^2 h^2}{3(1-\sigma^2)}N(m,\sigma) = 0$$

$$\frac{\lambda^3}{B^2}\frac{\partial}{\partial\alpha}B^2\frac{\partial t}{\partial\alpha} + \lambda^3 N(t,-\sigma) = 0$$

where λ^2 and λ^3 are introduced to make the terms of these equations have the dimensions of t. For cylindrical shells $\left(\frac{\partial\beta}{\partial\alpha} = 0\right)$

$$N(F) = \frac{1}{B}\frac{\partial}{\partial\beta}\frac{R}{B}\frac{\partial}{\partial\beta}\frac{1}{B}\frac{\partial}{\partial\beta}\frac{1}{B}\frac{\partial F}{\partial\beta} + \frac{1}{BR}\frac{\partial}{\partial\beta}\frac{1}{B}\frac{\partial F}{\partial\beta}$$

where R is the radius of curvature of the β-line. For non-cylindrical shells ($\partial B/\partial\alpha \neq 0$) the form of $N(F)$, in addition to the terms given above, contains the term

$$-\frac{1}{B}\frac{\partial}{\partial\beta}\frac{R}{B}\left(\frac{\partial}{\partial\alpha}\frac{1}{B}\frac{\partial B}{\partial\alpha}\right)\frac{\partial F}{\partial\beta}$$

For conical and cylindrical shells the system can be integrated approximately in the form of a series involving trigonometric and Bessel's functions, provided certain restrictions on the lengths of the shells and on the generatrix angle are imposed.

Several results obtained in the author's two earlier papers [*Prikl. Mat. i Mekh.*, 1945, vol. 11, no. 6; and 1946, vol. 10, no. 3], dealing with thin shells of zero Gaussian curvature which are so stressed that the state of stress can be decomposed into a momentless state and into a state produced by moments and boundary effects, appear as special cases in this more general treatment.

I. S. Sokolnikoff, USA

Buckling Problems

(See also Rev. 253)

255. Karl Marguerre, "On the application of the energy method to stability problems," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1138*, Oct. 1947, pp. 1–41 (transl. from *Jb. dtsch. VersAnst. Luftf.*, 1938, p. 252).

The first sections of the paper present a thorough discussion of the use of the principle of virtual displacements in the theory of elasticity. Criteria for equilibrium (stationary value of the total potential) and stability (in terms of the second variation of the total potential) are developed by means of this principle. The connection between these criteria and the usual energy methods of investigation of stability is logically established and illustrated by means of the classical example of the Euler column.

An advantage of this approach is that some of the characteristics of the behavior beyond the buckling load may be studied with the same procedures used before buckling, without recourse to additional considerations of large deflections. The development also allows the solution of problems (outside the range of the usual energy methods) which are basically nonlinear even in the stable region. As an example, the solution is outlined for the case of a slightly curved bar under a concentrated transverse load, with pin joints at both ends permitting no axial displacements.

Bruno A. Boley, USA

256. G. F. Carrier, "On the buckling of elastic rings," *J. Math. Phys.*, July 1947, vol. 26, pp. 94–103.

This paper deals with the problems of buckling and postbuckled deformation of a ring. The problem of buckling of a ring subjected to forces which initially produce no bending is reduced to the solution of an integral equation. This equation relates "fictitious" forces, proportional to the deformation, to the deformation itself by means of "influence functions" obtained by linear theory.

The Elastica problem for a circular ring under uniform normal pressure is solved in terms of elliptic functions, and numerical results are obtained. The stability problem for a circular ring with finite deflection loaded by normal forces of a special kind is resolved by the introduction of a perturbation of the loading. However, typographical errors, which are rather numerous in this report, prevent a complete understanding of the precise method followed. This technique, which is of great value in experimental work, is worthy of attention.

Finally, the dynamic stability of a circular ring, subject to a uniform normal pressure which varies sinusoidally with the time, is reduced to the same mathematical form as that which arises in the treatment of the similar problem for the column.

W. S. Hemp, England

257. Joseph Marin, "Creep deflections in columns," *J. appl. Phys.*, Jan. 1947, vol. 18, pp. 103–109.

The deflections of eccentrically loaded pin-ended columns are considered for columns made of materials that creep with time when subjected to constant load. The differential equation for the deflection is developed for materials having creep based on the log-log creep-stress law. Two solutions of the equation are offered, one obtained by a trial-and-error method involving difference equations, and one found by a numerical integration method.

The maximum column deflection and stress are determined as functions of time for the special case in which the exponent of the creep-stress law is equal to unity. The theoretical deflections are compared with experimental data obtained from tests on aluminum-alloy columns whose creep-stress exponent is near unity. Correlation between theoretical and test results is satisfactory, and it is pointed out that the critical load required to produce large deflections after a lapse of time may be lower than the usual elastic buckling load.

M. V. Barton, USA

258. J. P. O. Silberstein, "Plywood panels in end compression: curved panels with grain at various angles to the generators," *Austral. Counc. Aero.*, Jan. 1947, no. 31, pp. 3–25.

The critical buckling stress, p_{cr}, of a curved plywood panel is expressed in this report by the formula, $p_{cr} = \lambda_c \times$ (flexural rigidity)$/b^2t$, where λ_c is a dimensionless parameter, b is the width of the panel measured along its curved edges and t is its thickness.

From tests on three types of curved panels it was found that the parameter λ_c could be expressed by the equation $\lambda_c = \lambda' + \lambda_\infty$, where λ_∞ is the buckling parameter of a flat panel having the same dimensions as the curved panel and λ' is a function of the argument b^2/Rt, R being the radius of curvature of the panel.

Empirical formulas for λ' as a function of its argument were obtained for each of the three types of panels tested, and design curves were drawn for λ' for each type. These are intended to be applicable to panels whose imperfections are of about the same type and order of magnitude as those of the panels tested, as judged by variations in the radius of curvature. Since most of the panels tested were of the same thickness, the dependence of the parameter λ' on the thickness could not be completely established. H. W. March, USA

259, 260. **Ernesto Stagni, "A numerical method to determine critical loads in straight pin-ended members (Strutture lineari caricate di punta: un metodo numerico per la determinazione dei carichi critici)," *G. Gen. civ.*, Feb. 1947, vol. 85, pp. 82–95; "Determination of critical loads beyond the elastic limit in straight pin-ended metal structures (Strutture lineari metalliche caricate di punta: la ricerca del carico critico oltre il limite di proporzionalita)," *G. Gen. civ.*, June 1947, vol. 85, pp. 264–272.**

A straightforward numerical integration employing finite differences is applied to the buckling of columns. The process is applicable to columns of varying cross section, columns with axial loads applied at intermediate points, and statically indeterminate compound compression members (Vierendeel frames). The first part deals with elastic columns, the second with columns stressed beyond the elastic limit.

Basic energy and differential equations for buckling of columns are presented. Integration by means of finite differences lends itself to various approaches to the solution, notably Ritz's method and successive approximations. Accuracy is improved by a simple correction factor based on Taylor's series.

This paper covers primarily practical aspects; formulas are borrowed without proof from a more theoretical treatise on numerical integration. Von Kármán's method, as incorporated in German railway specifications, is used for columns stressed beyond the elastic limit. The examples contain numerous typographical errors. J. H. Meier, USA

261. **V. Rojansky and R. A. Beth, "Camptograms for beams in compression," *J. appl. Mech.*, Sept. 1947, vol. 14, pp. 202–208.**

Certain problems in dynamics may be analyzed by means of phase trajectories, in which the motion of a particle is represented by curves in which displacement is the ordinate and velocity is the abscissa. The authors have applied this topological method to obtain a graphical representation of the differential equation of an axially compressed uniform beam carrying a uniformly distributed lateral load. For this particular loading it is shown that a plot of the bending moment M against a parameter proportional to dM/dx is an arc of a circle.

A graphical procedure based on this concept is developed and illustrated by examples which include beam columns with concentrated transverse loads, stepwise variation in distributed load or in flexural rigidity, and with clamped or hinged ends. The authors call this graphical construction a "camptogram." Dana Young, USA

262. **I. P. Kuntze, "Stability of compressed plates satisfying Prager's theory of plasticity (Stabilité des plaques comprimées satisfaisant à la théorie de plasticité de Prager)," *C. R. Acad. Sci. URSS*, Feb. 20, 1947, vol. 55, no. 5, pp. 387–389.**

An earlier form of this reviewer's theory of plasticity [*Rev. Fac. Sci. Univ. Istanbul* (*A*), 1941, vol. 5, pp. 215–226] is applied to the plastic buckling of rectangular plates under edge thrusts. The following two cases are discussed: cylindrical buckling of a simply compressed plate, and buckling of a simply supported plate which is subjected to a constant edge thrust all along its edge.

The results reflect the special stress-strain relation adopted in the earlier form of the theory. A more general form has been developed recently and applied by G. H. Handelman and this reviewer to the plastic buckling of rectangular plates under edge thrusts; this will be published in a forthcoming *Nat. Advis. Comm. for Aero. Tech. Note.* W. Prager, USA

263. **W. Freiberger, F. S. Shaw, J. P. O. Silberstein, and R. C. T. Smith, "Plywood panels in end compression: flat panels with grain at various angles to direction of loading," *Austral. Counc. Aero.*, Jan. 1947, no. 30, pp. 3–35.**

This summarizes experimental and theoretical work on flat rectangular plywood panels in end compression. All panels had three plies of equal thickness. All panels were tested with loaded edges clamped and the other edges simply supported. For the panels of standard plywood with the direction of loading parallel to the grain of the face plies, the buckling loads were found to be in satisfactory agreement with previously obtained theory.

For panels of standard plywood with the load at 45 deg with the grain of the face plies, a theory is derived which is based on the differential equation of buckling and boundary conditions for the infinitely long panel and for the infinitely wide panel; from the results of tests a design curve is drawn between these limits for intermediate cases. Comparison is made with results obtained by another method by the reviewer [U.S. For. Prod. Lab. Rep. no. 1316, 1942]. The reviewer's results for 45-deg panels with simply supported loaded edges seem to have been compared with the authors' results for clamped loaded edges.

One group of panels had the grain of the face plies at 45 deg with that of the core ply, and was tested with the load at equal angles with the two grains. A theory is worked out for this case by an energy method, and is found to be reasonably well confirmed by tests.

In the experimental work the buckling load was determined both by the Southwell method as modified by Lundquist, and by finding the point at which there is a sharp change in slope of the "load vs. over-all compressive strain" curve. The authors recognize that the first method is not reliable for panels. In all cases the post-buckling behavior was studied, and for each type of panel curves are given in which the ratio of average stress to buckling stress is plotted against the ratio of edge stress to buckling stress. In this connection reference may be made to the work of C. B. Norris and A. W. Voss on the effective width of plywood panels [U.S. For. Prod. Lab. Rep. no. 1316E, 1943, and no. 1316I, 1945]. H. W. March, USA

264. **A. A. Ilyushin, "The elasto-plastic stability of plates," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1188*, Dec. 1947, pp. 1–30 (transl. from *Prikl. Mat. i Mekh.*, 1946, vol. 10, p. 623).**

In this paper the author is concerned with the elastic-plastic stability of plates, which in their state of neutral equilibrium may possess purely elastic regions, purely plastic regions and elasto-plastic regions (where only part of the thickness is plastic). He restricts himself mainly to compressed plates in which "before buckling" regions of elasto-plastic strains are missing.

After buckling the originally elastic region of the plate will be elastically deformed, whereas the originally plastic region will be resolved into subregions, one of which remains purely plastic and the other of which becomes elasto-plastic. The boundary of these two latter regions of course appears as an unknown in the problem,

which (if treated in a strict way) gives rise to the solution of two simultaneous nonlinear partial differential equations. In the approximate analysis given, the problem is reduced to a single linear equation of the Bryan type.

The basic elastic and plastic equations of the theory are represented by:

$$X_x - \frac{1}{2} Y_y = \frac{\sigma_i}{e_i} e_{xx}, \quad Y_y - \frac{1}{2} X_x = \frac{\sigma_i}{e_i} e_{yy},$$

$$X_y = \frac{\sigma_i}{3e_i} e_{xy}, \quad e_{xx} + e_{yy} + e_{zz} = 0 \ldots\ldots [1]$$

where X_x--, e_{xx}-- are the stresses and strains, and σ_i and e_i are defined as:

$$\sigma_i = \sqrt{X_x^2 + Y_y^2 - X_x Y_y + 3X_y^2},$$

$$e_i = \frac{2}{\sqrt{3}} \sqrt{e_{xx}^2 + e_{yy}^2 + e_{xx}e_{yy} + \frac{1}{4} e_{xy}^2} \ldots\ldots [2]$$

In the elastic region σ_i must be put equal to Ee_i. Beyond the elastic limit σ_i must be looked upon as a function of e_i, which has to be determined in advance for each individual material. It appears from both Equations [1] and [2] that the material is considered subject to the condition of constant volume in every state of stress.

The paper is divided into three sections. In the first the author derives expressions for the forces and moments in terms of the strains in the middle surface of a shell. In the second he sets up a general theory for the stability of compressed plates. In the third section the author discusses some special cases, including: the stability of a plate of arbitrary shape under equal biaxial compressions; buckling into cylindrical shape of a rectangular plate under uniaxial stress or strain; and a plate in a uniform state of stress before buckling. C. B. Biezeno, Holland

Joints and Joining Methods

(See also Rev. 272)

265. G. Sachs, "Note on the tightness of expanded tube joints," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 285–286.

The tightness of expanded tube joints is re-evaluated in terms of a simplified analysis. A modification of the maximum shear stress criterion of yielding is assumed, and the residual stress between tube and plate is derived as $p_b(1-a^2/b^2) - k_t(a^2/b^2)\ln(b/a)$ where p_b = forming pressure between tube and plate, k_t, k_p = yield strength of tube and plate material, and a, b = inside and outside radii of the tube respectively.

It is concluded that if $k_t = k_p$ and b/a is between 1.3 and 1.5, the maximum residual pressure may be $0.17k_p$; for $k_t = 2k_p$ and $b/a = 1.65$, the maximum residual pressure may be $0.30k_p$.

William Schroeder, USA

266. Samuel J. Rosenfeld, "Analytical and experimental investigation of bolted joints," *Nat. Advis. Comm. for Aero. Tech. Note No. 1458*, Oct. 1947, pp. 1–48.

This paper is concerned with the symmetrical butt-strap joint with evenly spaced bolts in line with the axial force. Three solutions of the problem are obtained:

1 On the basis of an earlier report [*Nat. Advis. Comm. for Aero. Tech. Note No. 1051*, 1946] a recurrence formula is developed which together with the boundary conditions at the ends of the plate result in a system of first-degree simultaneous equations in which the bolt loads are the unknowns.

2 By observing that the above recurrence formula is in effect a second-degree finite-difference equation with constant coefficients, and solving it as such.

3 By replacing the bolts by elastically equivalent cemented layers between plate and straps and applying to it shear-lag theory.

All three methods of calculation are shown to be in close agreement. The remainder of the paper deals with the testing of a number of butt-strap joints and a comparison, which the author describes as "generally satisfactory," of the test results with the theory. F. Hymans, USA

Structures

(See also Rev. 246)

267. Hans Spiller, "Recent methods of calculating continuous beams and frames (Neuere Methoden zur Berechnung kontinuierlicher Trager und Rahmengebilde)," *Bauplan. u. Bautech.*, Dec. 1947, vol. 1, pp. 183–192.

Except that the word "recent" in the title is misleading because the methods discussed are at least fifteen years old, the paper gives a good presentation and a comparison of the Hardy Cross and the slope-deflection procedures for the analysis of continuous beams and rectangular frames. A later paper is to treat varying moments of inertia and frames having sloping members.

Joseph S. Newell, USA

268. Joseph Kempner, "Recurrence formulas and differential equations for stress analysis of cambered box beams," *Nat. Advis. Comm. for Aero. Tech. Note No. 1466*, Oct. 1947, pp. 1–50.

An analysis is given of the stresses due to bending in two types of prismatic cambered box beam, with idealized cross sections, having stringers which carry normal stresses and webs which carry shear stress only: (1) A closed section of doubly symmetrical hexagonal shape with a stringer at each corner; (2) an open section obtained by removing the bottom surface of the hexagonal section.

The solution is expressed as the sum of primary stresses taken from elementary bending theory and secondary stresses. Because of the simplicity of the idealized cross section, the transverse distribution of secondary stresses may be expressed in terms of a single quantity. The spanwise variation of this quantity is governed by a second order differential equation, or a system of linear algebraic equations having a continuant matrix of coefficients. Solutions by both methods are compared with experiment and show good agreement.

Stanley U. Benscoter, USA

269. A. Albrecht, "The compound girder as combination of steel girder with reinforced concrete slab (Der Verbundträger als Kombination von Stahlträger mit Eisenbetonplatte)," *Ingenieur, 's Grav.*, Nov. 7, 1947, vol. 59, pp. Bt81–88.

The paper reviews work done, chiefly in Switzerland in the past 15 years, on compound beam constructions, in which steel girders and reinforced concrete slabs are joined together to form an integral load-carrying structure. The resulting construction combines the stiffness and low maintenance cost of concrete structures with the easy erection, lighter weight and better reinforcing possibilities of steel structures.

The principal problem in the design of such compound girders is the transfer of longitudinal shear from girder to slab, which is best accomplished by angles welded to the steel girders and serving as dowels. The paper reports the results of a series of tests of such dowels on specially prepared shear specimens, and of static

and fatigue tests on complete small-span compound-girder constructions. Numerical examples of calculation are given, with applications to building and bridge constructions, including a bibliography on the subject. M. Hetényi, USA

270. C. E. Andrew, "Unusual design problems—second Tacoma Narrows bridge," *Proc. Amer. Soc. civ. Engrs.*, Dec. 1947, vol. 73, pp. 1483–1497.

The author describes the corrective measures applied to the first Tacoma Narrows Bridge, prior to failure. The mechanism of the instability leading to the failure is discussed and an outline of the research required to formulate a satisfactory design criterion for the second bridge is given. This includes a discussion of the problems associated with aerodynamic and elastic properties of the structures and a description of wind-tunnel model studies. The troublesome problems arising in the construction of an aerodynamically and dynamically similar bridge model are pointed out. Some details of the final design of the second bridge are given. R. L. Bisplinghoff, USA

271. J. C. Schönfeld, "An electric model for the experimental determination of the deformation of a statically indeterminate frame (Een elektrisch model voor het experimenteel bepalen van de vervorming van een statisch onbepaalde vlakke vak- of raamwerkconstructie)," *Ingenieur, 's Grav.*, Sept. 26, 1947, vol. 59, pp. 0–43 to 0–51.

Employing choke coils, transformers, and condensers, the author develops an electrical model of a straight bar in which the terminals represent the ends of the bar, the currents are proportional to the forces and moments impressed on the end sections, and the potentials of the terminals (which are the measured quantities) are proportional to the linear and angular displacements of the end sections. A frame is represented by an assembly of a number of such models with their terminals connected according to the position and boundary conditions of the bars in the frame. Two illustrative examples are given.
F. Hymans, USA

272. R. W. Stewart, "Analysis of frames with elastic joints," *Proc. Amer. Soc. civ. Engrs.*, Dec. 1947, vol. 73, pp. 1499–1506.

The author's "traverse method" of analyzing continuous frames [*Trans. Amer. Soc. civ. Engrs.*, 1939, p. 521] is here used to determine the moments when joints are given assumed elastic properties. As in other slope and deflection analyses the traverse method gives deflections with the same convenience as moments.
L. E. Grinter, USA

Plastic Flow, Failure

(See also Revs. 219, 220, 257, 262, 264, 265, 292, 298, 303, 304)

273. C. Torre, "Influence of the middle principal stress on the condition of flow and failure (Einfluss der mittleren Hauptnormalspannung auf die Fliessund Bruchgrenze)," *Ost. Ingen.-Arch.*, vol. 1, nos. 4 & 5, pp. 316–342.

After a review of the literature, and a discussion of the stress equilibrium conditions, the author defines the "kind" of loading (Belastungsart) in terms of the principal stress differences. The limiting condition of failure $F(\sigma_1, \sigma_2, \sigma_3) = 0$ is then expressed in terms of the maximum and minimum principal stresses σ_1 and σ_3, together with a parameter which takes into account the effect of the mean principal stress. The limiting curve may also be expressed in terms of $p_m = (\sigma_1 + \sigma_3)/2$ and $\max \tau = (\sigma_1 - \sigma_3)/2$.

It is shown that, in general, to represent the condition of failure, several envelope curves are required in the σ,τ plane instead of a single envelope as given by the Mohr theory. The relation between the envelope curve $\tau = \tau(\sigma)$ and the curve $\max \tau =$ function of p_m is discussed. A surface in terms of a paraboloid of revolution is shown as a general representation of certain theories of failure. Limiting curves in the σ_1,σ_3 plane and in the σ,τ plane, based on various tests reported in the literature, are presented and discussed with particular reference to the effect of the mean principal stress. A. M. Wahl, USA

274. G. H. Handelman and W. Prager, "Stress-strain relations for incompressible plastic materials with strain hardening" (in English), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Mar.–Apr. 1947, vol. 11, pp. 291–292.

The material is first assumed to be quasi-elastic, the deviatoric components of strain remaining proportional to the components of the stress deviation. If such a material undergoes changes of stress for which the octahedral shearing stress remains constant, the stress-strain relations for loading and unloading would yield different strains, which the authors consider as unlikely.

They show that this difficulty does not arise if a differential stress-strain relation is assumed, in which the increments of the components of plastic strain are proportional to the components of the stress deviation. P. P. Bijlaard, Holland

275. J. A. Pope, "The apparent yield strength of plain carbon steel," *Engineering*, Sept. 19, 1947, vol. 164, no. 4260, pp. 284–288.

The author presents his explanation of the discrepancy between the yield point observed in simple tension or compression and the yield point computed from tests in torsion, in bending, and in combined tension and bending. On the basis of tests by J. R. M. Morrison [*Proc. Instn. mech. Engrs.*, Jan. 1940, vol. 142, p. 193] the author computes the thickness of a surface layer of the material which he considers to resist stresses higher than the tensile or compressive yield point. He associates the thickness of the layer with the size of the specimen but not with the stress gradient. William R. Osgood, USA

276. Maxwell Gensamer, Edward Saibel, J. T. Ransom, and R. E. Lowrie, "Report on the fracture of metals, Parts I, II," *Welding Res. Supplement*, Aug. 1947, vol. 12, pp. 443–484.

This survey of present knowledge of the laws and mechanism of fracture was requested by the Bureau of Ships as a basis for a research program. Results are expressed in terms of flow and fracture strengths, and the effects on these strengths of stress combination, strain and strain anisotropy, nonuniform and cyclic stressing, temperature, strain rate, size, composition and metallurgical structure are discussed. In the light of this discussion, the notched-bar impact test and the problem of ship cracking are examined, followed by a survey of the theories of fracture (cohesive strength theory, microcrack theory, modified Griffith theory, melting theory, Furth's theory).

The following experimental program is proposed: (1) Separate the effects of strain and resulting strain anisotropy upon resistance to deformation under combined stresses. (2) Separate the effects of state of stress, strain and the resulting strain anisotropy upon the resistance to fracture under combined stress at brittle temperatures. (3) Effects of prior strain history upon the resistances to both deformation and fracture. (4) Effects of temperature on both these resistances under combined loads, at very low strain rates. (5) Effects of rate of straining on both these resistances under combined stresses, at a series of constant temperatures. (6) Effects of composition on the type of structure, on precipitation phenomena and on solid solution alloying phe-

nomena. (7) Effects of cyclic loading (*a*) under combined loads, upon deformation and fracture properties, (*b*) upon brittleness, (*c*) effects of prior treatments upon these results of cyclic loading. (8) Effects of stress and strain gradients upon the resistances to deformation and to fracture. (9) A theoretical study of fracture is needed in conjunction with this experimental program, based on: (10) Experimental investigation of the nature of flaws and micro-defects in metals. (11) Behavior of the fracture strength in triaxial tension as a test of the microcrack theory of fracture. (12) Local ductility of iron and steel when broken at low temperatures. (13) Effect of distribution of deformation upon resistance to fracture. Leon Beskin, USA

277. Sir Geoffrey Taylor, "A connection between the criterion of yield and the strain ratio relationship in plastic solids," *Proc. roy. Soc., London, Ser. A*, Dec. 3, 1947, vol. 191, pp. 441–446.

This paper refers to experimental investigations of the plastic stress-strain ratio relationship by Lode in 1926, and by the author and the late H. Quinney in 1931. These investigators observed the relation connecting the two stress and strain variables defined by:

$$\mu = \frac{2\sigma_2' - \sigma_1' - \sigma_3'}{\sigma_1' - \sigma_3'}, \quad \nu = \frac{2e_2' - e_1' - e_3'}{e_1' - e_3'} \ldots\ldots [1]$$

(In these expressions the author has substituted for the usual principal stresses and strains the "deviators" of stress and strain σ_1', e_1' etc., that is, the principal stresses and strains minus their average values.)

Next the author refers to a recent paper by Hill, Lee, and Tupper. In this paper it was shown that if the criterion of yield is taken in the form

$$(\sigma_1')^2 + (\sigma_2')^2 + (\sigma_3')^2 = \frac{2}{3} Y^2 \ldots\ldots [2]$$

where Y is the yield stress in simple tension, then the work done during a given small strain is a maximum if μ and ν have the relation

$$\mu = \nu \ldots\ldots [3]$$

Since the above mentioned investigations indicated a somewhat different relation between μ and ν, the author calculates what the criterion of yield would be to make the work done during a small strain a maximum when μ and ν have their experimentally determined relationship, instead of the relation [3]. Using the tension-torsion tests on copper which he and Quinney made in 1931, he compares the results of these tests with three theoretical curves. These curves were derived from: (*a*) The criterion of yield given by Equation [2]; (*b*) a criterion of yield corresponding to the maximum work during small strain when the relation between μ and ν is as observed in the tests; (*c*) the maximum shear stress criterion. He finds that curve (*b*), corresponding to the new criterion, comes very close to the curve (*a*), based on Equation [2]. The experimental points are closely grouped around these two curves. A. Nadai, USA

278. H. A. Elliott, "An analysis of the conditions for rupture due to Griffith cracks," *Proc. phys. Soc. London*, Mar. 1947, vol. 59, pp. 208–223.

The author's purpose is to extend the Griffith theory of rupture through a more detailed consideration of interatomic forces which resist the spread of a crack. He assumes that $f(z)$, the interatomic force, is the same function of z as the normal component of stress in the z-direction is the displacement in the z-direction for planes equidistant from the plane of the crack. The latter is obtained from the solution of the problem of the stress around a crack in an elastic solid subjected to a tension, with no stresses acting across the surface of the crack. This leads to a law of force which resembles that of interatomic forces. The initial rate of increase of stress with strain is E, there exists a maximum force, and the stress dies away to zero for infinite strain.

The theory is compared with the experiments of Griffith and fair agreement is found. Edward Saibel, USA

279. F. R. N. Nabarro, "Dislocations in a simple cubic lattice," *Proc. phys. Soc. London*, Mar. 1947, vol. 59, pp. 256–272.

The properties of ideal linear dislocations are investigated more systematically than heretofore. A large gap still exists between an understanding of the properties of these ideal linear dislocations and the mechanical properties of real metals.

Clarence Zener, USA

280. P. Rehbinder, "New physico-chemical phenomena in the deformation and mechanical treatment of solids," *Nature, Lond.*, June 28, 1947, vol. 159, pp. 866–867.

The presence of surface active media, such as oleic acid or cetyl alcohol, is reported to have an unexpected influence upon the mechanical behavior of metals. Thus the number of slip lines in a tin crystal subject to a given extension is said to increase fifteenfold. Again, the presence of such media is said to decrease the yield stress of previously deformed tin to only one half of the original value.

It is believed that the surface active medium penetrates along the slipbands, thereby lowering their resistance to further slip. Particular emphasis is placed upon the importance of such "internal lubrication" in all problems involving the machining and working of metals. Clarence Zener, USA

281. Nelson W. Taylor, "Mechanism of fracture of glass and similar brittle solids," *J. appl. Phys.*, Nov. 1947, vol. 18, pp. 943–955.

Fracture is described as the result of the stretching of the bond between atoms by a certain amount. The bond stretched this amount, however, has greater unit strain than the average value thereof, because thermal vibrations cause constant redistribution of the total elongation of a given length of fiber. The author concludes that a critical stress-time relationship for fracture exists, and attempts to rationalize the formula for this relationship developed by Glathart and Preston. Further supporting data by Anderegg are submitted.

Mechanisms by which chemical environment may affect the life at a given stress are discussed, along with the need for more research on this subject.

To account for data by Powell and Preston, the hypothesis is made that a compressive stress increases the strength under tensile stress in another direction. In addition, the effect of initial stress, and the statistical distribution and possible preferred orientation of flaws is discussed. F. H. Fowler, Jr., USA

282. R. Hill, E. H. Lee, and S. J. Tupper, "The theory of combined plastic and elastic deformation with particular reference to a thick tube under internal pressure," *Proc. roy. Soc., London, Ser. A*, Nov. 18, 1947, vol. 191, pp. 278–303.

The authors assume a material without strain hardening and behaving as a so-called Prandtl-Reuss body, in which the increments of the components of the plastic deviatoric strain tensor are at any moment proportional to the components of the stress deviation. The elastic strain in the plastic zones is taken into account. The relationship between stress and strain deviation is represented in the reduced stress-strain diagram, suggested by

Prager. If yielding is governed by the shearing energy the flow-limit figure in the diagram is a circle. Postulating a maximum energy dissipation, any given flow-limit figure determines the mechanism of plastic flow and vice versa.

The stress and strain variation in a long thick tube under internal pressure and with zero longitudinal extension is represented in the diagram for all stages of deformation. The simultaneous differential equations for the plastic flow are integrated numerically by a step-by-step triangulation along the characteristics for particular stages in the flow and particular elements.

The longitudinal stresses in the plastic region appear to be smaller than those given by earlier solutions in which the elastic strains were neglected, the maximum error being over 60 per cent. The authors assume continuous flow and do not take into account the fact that actual yielding propagates discontinuously in discrete flow layers with interjacent elastic material.

P. P. Bijlaard, Holland

283. C. Gurney, "Delayed fracture in glass," *Proc. phys. Soc. London*, Mar. 1947, vol. 59, pp. 169–185.

The author discusses several processes which might account for the phenomenon of delayed fracture in glass. One discussion is devoted to showing that the Griffith theory is not inconsistent with the gradual growth of a crack. However, this requires a certain co-operation among neighboring atoms, and the probability of this occurring is small.

Another possibility is the approach to homogeneous equilibrium under stress. This involves the postulation of a high-temperature phase and a low-temperature phase in glass, and the assumption that after manufacture the approach to equilibrium ceases because of the high internal viscosity. Stress reduces the internal viscosity and enables the approach to equilibrium to proceed. Changes take place much faster at the root of the crack because of the stress concentration, causing the crack to spread.

The third possibility is the approach to heterogeneous equilibrium. This may manifest itself through evaporation at the ends of the crack, or through atmospheric attack of the glass. The former is estimated to be quite small and unimportant. The latter effect is most important at the root of the crack where the material is more chemically active because of the concentration of strain energy. At this point a complex of glass and atmospheric constituents is formed. The crack spreads continually if the strength of this complex is less than the load imposed on it.

Edward Saibel, USA

284. H. Brandenberger, "New tests in the field of materials research (Neue Ergebnisse auf dem Gebiete der Materialforschung)," *Schweiz. Bauztg.*, Sept. 13, 1947, vol. 65, pp. 509–515.

This paper deals with the extension of the theory of "space and lattice stresses" proposed and discussed in earlier papers by the author. The theory is now applied to certain aspects of plastic flow, particularly in connection with alternating tension and compression beyond the yield point.

The Bauschinger effect is explained in terms of these stresses and a means of calculating the fatigue strength from the yield point is developed, which gives remarkable agreement with the endurance limit for certain steels. In addition, the author's theory is found to support the maximum strain (St. Venant) criterion for fracture of ductile metals.

Louis F. Coffin, Jr., USA

285. T'ing-Sui Kê, "Stress relaxation across grain boundaries in metals," *Phys. Rev.*, July 1, 1947, vol. 72, pp. 41–46.

The paper is a contribution toward further elucidation of the concept of relaxation of shear stress across grain boundaries in metals. Specimens of 99.991 per cent pure aluminum were tested. The temperature dependence of the internal friction and the rigidity modulus were measured as a function of frequency of torsional vibration and as a function of grain size.

For the same specimen, an increase of frequency of vibration shifted the internal-friction curve and the rigidity relaxation curve to higher temperatures. Keeping the frequency of vibration constant and varying the grain size, it was found that increase in grain size had the same effect as an increase in the frequency of vibration.

The observed internal friction and rigidity relaxation could be expressed as functions of the parameter $(G.S.) \times f \times exp(H/RT)$, where $(G.S.)$ is the grain size or average grain diameter of the specimen, f is the frequency of vibration, and H is the heat of activation. It is shown that all the observed phenomena are necessary manifestations of the stress relaxation across grain boundaries and are in strict accordance with the concept of the viscous behavior of the grain boundaries in metals.

Hans F. Winterkorn, USA

286. A. S. Nowick and E. S. Machlin, "Dislocation theory as applied by NACA to the creep of metals," *J. appl. Phys.*, Jan. 1947, vol. 18, pp. 79–87.

By assuming a specific mechanism for the generation of dislocations, an equation for the steady-state creep rate of pure annealed polycrystalline metals is derived, which has the same form as empirical relations which have been used to correlate creep data. It is assumed that creep takes place by the motion of dislocations, and the creep rate is shown to depend on the rate of generation of dislocations.

The formation of a dislocation involves the limitation of the direction of motion of a group of atoms, which results in a large negative entropy of activation for the process. The resulting equation for rate of creep involves temperature, stress, material constants, atomic and lattice dimensions, and a "back stress" term which is connected with work hardening, but which has not yet been evaluated. Order of magnitude calculations are in good agreement with data in the literature and seem to indicate that the dislocations are only one atom long. This material was originally presented in *Nat. Advis. Comm. for Aero. Tech. Note No. 1039.*

Stephen H. Crandall, USA

287. C. W. MacGregor, N. Grossman, and P. R. Shepler, "Correlated brittle fracture studies of notched bars and simple structures," *Welding Res. Supplement*, Jan. 1947, vol. 12, pp. 50–56.

The factors determining ductile or brittle behavior are given as: (1) material, (2) temperature, (3) velocity of deformation, and (4) constraint or relation of the three principal stresses. Within a limited range the quantitative relation between velocity and temperature is expressed as $\ln V = C - Q/RT$, where V is deformation velocity, C a material constant, Q the heat of activation of the material, R the molar gas constant, and T absolute temperature. If two different shapes of specimens are made from the same material and tested at the same velocity, the authors consider the constraint or "constraint effect" to be the same if the ductile-brittle transition temperature is the same. They also conclude that if the deformation velocity is changed for these specimens, the transition temperature will change according to the above formula.

Tests on specimens of two shapes are described: (1) the freely supported concentrically loaded circular disk in plate bending, (2) the notched bar (semicircular notch with 1/8-in. radius) in simple bending. To obtain the same "constraint effect" the depth of the notch in the bar was varied until the same ductile-

brittle transition temperature was obtained at the same deflection rate as used for the circular disk. Tests were also made at other deflection velocities, and these confirm the authors' conclusions over the limited range of deformation rates. The effect of the direction of rolling on transition temperature is found to be small.

In conclusion the authors consider how the designer may check a structure or machine part for safety against brittle failure if he knows the operating temperature and expected rate of loading in service. M. J. Manjoine, USA

288. **R. Garber, "The mechanism of calcite and nitre twinning under plastic deformation"** (in English), *J. Phys. Acad. Sci. USSR*, 1947, vol. 11, No. 1, pp. 55–66.

Twins are formed only under concentrated loads, i.e., they presumably require very high local stresses. Otherwise, fracturing by cleavage occurs.

Up to a certain (concentrated) load, narrow lens-shaped "elastic twins" appear, which usually disappear again on removal of the load. Then, under a rather high load, a permanent twinning nucleus extending over the specimen width develops. Subsequently, the twinned portion of the crystal can be enlarged by a comparatively small and distributed load.

If a concentrated load is applied at a location outside of the twinned layer, a number of "polysynthetic" twins are formed in which twinned sections alternate with portions of the initial crystal. By means of Newton's rings it is shown that the surface of the initial crystal in the vicinity of an elastic twin is distorted, forming a depression at one side and an elevation at the other side of the twin, rather than consisting of displaced parallel planes. George Sachs, USA

Design Factors, Meaning of Material Tests

(See also Revs. 218, 227, 276, 284, 287, 294)

289. **H. E. Davies and J. McKeown, "The significance of mechanical testing,"** *Metallurgia*, Nov. 1947, vol. 37, pp. 19–22.

This summarizes two papers presented at a meeting of the Midland Metallurgical Societies. H. E. Davies discusses the lack of correlation between design and the usual tests performed to determine the acceptance of a material for service. J. McKeown reviews work on creep and fatigue at elevated temperature. In summary he states, "The effect of speed of stress or strain reversal is very important, and the life is dependent on the mean stress and if this is sufficiently high is determined by considerations of creep resistance." Frank J. Mehringer, USA

290. **R. M. Howarth, "Some elementary considerations of the stress-strain curve,"** *Aircr. Engng.*, Dec. 1947, vol. 19, pp. 372–377.

The author points out that if Hooke's law applies exactly to the true elastic stress-strain relation, the nominal stress-strain curve should be slightly curved, and vice versa, such deviations being of the order of one per cent for high-strength materials. He also discusses the phenomena of an apparent "double modulus" for some materials, as well as methods of defining yield or proof stresses. A. J. Durelli, USA

291. **S. Luthander, "The safety concept in strength of materials from the statistical viewpoint (Det hallfasthetstekniska sakerhetsbegreppet ur statistisk synpunkt),"** *Tekn. Tidskr.*, Oct. 18, 1947, vol. 77, no. 42, pp. 769–780.

In this paper the author makes an attempt at a rational definition of the factor of safety in structures. The discussion is focused chiefly on aircraft structures, and the mechanism of failure under consideration is fracture in fatigue under the combined action of *amplitude* and *mean value* of stress.

Based on a study of the available experimental information on aircraft structures, certain normal patterns of behavior under fatigue are established. The premises for establishing the statistical spread in the strength figures, as well as the statistics of load conditions, are necessarily more speculative. Based on adequate information in these respects, however, the author applies the statistical reasoning established by A. Palmgren ["Die Lebensdauer von Kugellagern," *Z. Ver. dtsch. Ing.*, 1924, vol. 68, p. 339] for a determination of the life duration of a structure.

The result is expressed in the form of relations between the life of a structure and its factor of safety.

C. Richard Soderberg, USA

292. **J. B. Friedmann, "Generalized mechanical properties of solid bodies"** (in English), *C. R. Acad. Sci. URSS*, Mar. 30, 1947, vol. 55, no. 9, pp. 817–820.

This article presents results of tests on duralumin, high-strength Al-Zn-Mg alloys, austenic Cr-Ni steels of 18-8 type and high-strength steel of the chromansil type.

By plotting the "generalized" quantities of Ludwig, maximum shear stress against maximum shear strain, the author endeavors to show that the experimental points for several modes of loading (tension, compression, torsion, double shear) can be represented by a single curve for any one material. This is successfully done for all the materials tested except the high-strength steels, which show consistent anomalies between the tension and torsion loadings.

For all but these latter materials the "generalized shearing strengths" (the value of the maximum shear stress at shearing rupture) are practically independent of the mode of loading

Stephen H. Crandall, USA

Material Test Techniques

(See also Revs. 236, 299)

293. **A. W. Nolle, "Acoustic determination of the physical constants of rubberlike materials,"** *J. acoust. Soc. Amer.*, Jan. 1947, vol. 19, pp. 194–201.

In general, the effective elastic modulus and internal resistance (due to viscosity) of rubberlike materials are functions of frequency. By measurement at a single frequency of the decrease in amplitude and the change in phase of a progressive longitudinal wave traveling along a thin strip of material, the elastic and resistive coefficients may be calculated.

This paper derives the necessary relations and describes the apparatus used. The vibration driver and detector elements are respectively a crystal disk recorder and a pickup head, the latter bearing lightly against the stretched strip. The measurements reported on a sample of synthetic elastomer cement show that the phase velocity, Young's modulus and attenuation increase with frequency, while the resistance coefficient decreases.

Vincent Salmon, USA

294. **R. F. Hanstock, "Damping capacity, strain hardening, and fatigue,"** *Proc. phys. Soc. London*, Mar. 1947, vol. 59, pp. 275–287.

A new type of electromagnetic exciter, and a related test procedure for measuring damping capacity, are described for producing resonant torsional vibrations in specimens having a test portion 1 cm in diam × 10 cm long. Torsional strains were produced

of sufficient amplitude to cause fatigue failure in binary aluminum alloys (containing 5 per cent and 11 per cent of Mg) at frequencies near 1 kc per sec with a maximum amplifier output of 50 watts. Tests were made with a constant root-mean-square current output, rather than with constant strain, and measurements of damping capacity were periodically obtained by stopping excitation, and observing the rate of amplitude decay.

For these two alloys, a decrease in damping capacity and increase of amplitude occurs during vibration. Upon inception of a fatigue crack there is a sharp decrease in amplitude and frequency, and increase in damping capacity. The author suggests that fatigue failure occurs after a definite amount of strain hardening (irreversible strain), the life of a specimen being related to the irreversible strain imposed per cycle (the reviewer notes that this is in agreement with the theory proposed by Orowan). The fatigue life is not closely correlated with a definite dissipation of vibrational energy; a considerable proportion of the damping loss may appear as thermal energy. T. J. Dolan, USA

295. **H. Sulser, "Bursting test with a ball (Die Kugeldurchstossprüfung),"** *Schweiz. Arch.*, Feb. 1947, vol. 13, pp. 45–54.

A method is described for testing the strength of cloth and paper under conditions of equibiaxial tensile stress. Instead of inflating a circular membrane specimen by internal air or fluid pressure, as in the usual biaxial bursting test, the membrane is loaded to bursting by the driving of a punch in the shape of a half sphere. The friction which develops between sphere and specimen, and which increases the bursting force, can be eliminated by rotating the punch during the test.

The test, particularly its friction aspect, is mathematically analyzed. Comparative results of uniaxial tension tests and ball-bursting tests are given. A. M. Freudenthal, USA

296. **W. P. Welch, "A proposed new shock-measuring instrument,"** *Proc. of Soc. for Exp. Stress Anal.*, 1947, vol. 5, no. 1, pp. 39–51.

A shock-measuring instrument, composed of a mass, spring, and damping device, having one degree of freedom is proposed. The shock is defined in terms of the displacement. Application of the Westinghouse transient analyzer indicates an optimum viscous damping of 0.5 critical. George Lee, USA

Mechanical Properties of Specific Materials

(See also Revs. 269, 281, 283, 292, 294, 295)

297. **J. N. Greenwood and J. H. Cole, "Creep rate of various industrial leads,"** *Metallurgia*, Sept. 1947, vol. 36, pp. 233–235.

Creep of various industrial leads was measured at 20 to 24 C to determine how composition differences influenced creep. Eight leads were tested at tensile stresses of 350 and 500 psi. Tests at 350 psi lasted up to four years. The tests show that the total impurity content of industrial lead is no guide to its behavior under prolonged stress. It was also found that the influence of one impurity on the creep properties may be affected by the presence of others. Joseph Marin, USA

298. **George Halsey, "Nonlinear viscous elasticity and the Eyring shear model,"** *J. appl. Phys.*, Dec. 1947, vol. 18, pp. 1072–1097.

The author attempts to derive equations which would describe the experimental stress-strain-time relationships of textile fibers, by making use of an analogy with a model consisting of an energy-storage element (spring) paralleled by a spring in series with an energy-dissipation device (dashpot containing viscous fluid). This attempt is successful but the agreement is better when a substance with a yield point is substituted for the viscous fluid. Further improvement requires an assumption of complex behavior by the energy-storage and energy-dissipation devices. Benjamin Miller, USA

299. **J. R. Bressman, "Method for determining tensile properties of refractory materials at elevated temperatures,"** *J. Amer. ceram. Soc.*, May 1, 1947, vol. 30, pp. 145–152.

Short-time tensile tests and "stress rupture" tests were made on sillimanite refractory at temperatures up to 1950 F. The specimens used in both series of tests were a modification of the standard tensile specimen for porcelain. An elaborate and meticulous procedure was used to insure a minimum of bending stresses in the static tests. The material was tested in two conditions, as received and annealed at 1800 F for a half hour.

The maximum tensile strength occurred at 1800 F. The stress-rupture data while meager indicate a favorable comparison between the sillimanite refractory and high-temperature alloy steel for turbine rotors, on the basis of strength-density ratio. Frank J. Mehringer, USA

300. **L. E. Mong, "Elastic behavior and creep of refractory bricks under tensile and compressive loads,"** *J. Res., Nat. Bur. Stands.*, Feb. 1947, vol. 38, pp. 229–240.

In many uses of refractory materials it is important to keep the distortion within certain limits. This distortion may be made up of plastic and elastic deflection caused by external loading, including creep or plastic flow with time, and thermal distortion. This paper describes these distortions for nine brands of fuel brick for temperatures up to and including 950 C. The relative load-carrying capacities are compared at 950 C. Changes in the modulus of elasticity before and after heating or stressing are recorded. M. J. Manjoine, USA

301. **F. Jager, "The crushing of pulverizable materials (Le broyage des corps pulverulents),"** *Tech. mod.*, Nov. 1–15, 1947, vol. 39, pp. 357–359.

This paper deals with the crushing of several types of cements under slowly applied pressure and under impact. The author studies the efficiency of both methods and finds slow pressure more advantageous. Special consideration is given to the effect on the compressive strength of cement specimens of the amount of pressure used in crushing, a crushing pressure of about 10,000 kg/cm² giving the highest strength. A. J. Durelli, USA

302. **S. L. Dart and Eugene Guth, "Elastic properties of cork—II. Stress-temperature relationship of compressed cork; III. Hydrostatic and ordinary load-compression curves for cork,"** *J. appl. Phys.*, May 1947, vol. 18, pp. 470–478.

In the first part, as a necessary step in the complete definition of the authors' previous expression for the compressive stress in cork as a function of compression, temperature and time [*J. appl. Phys.*, 1946, vol. 17, p. 314], the authors present an experimental method for separating the two relations making up this function. They also establish an experimental formula for one of these relations, namely the stress as a linear function of temperature (which is essentially the load-compression curve of the cork). In their earlier work they developed the other relation, for stress variation with time and temperature, independent of compression.

These experimental relations are described in terms of modern theories. The time dependency expression finds agreement with a function produced by the generalized Boltzmann theory for the non-Hookean case. The theoretical stress-temperature relation is

formulated by the use of the energy equation and Maxwell relation of thermodynamics. Explanations are made on the basis of material structure.

In the second part, the authors use two experimental techniques, giving different S-shaped curves, and explain these curves on the basis of the cellular structure and of the two-phase structure of cork (corky material and the air trapped in the cork cells). The ordinary load-compression curve, which is similar in shape to the true stress-strain curve of a material which yields discontinuously in tension, has its component parts explained in terms of the physical phenomena occurring in the cork. An equation is presented for the latter part of the diagram. Elasticity theory is used to connect the two different types of relations obtained in the techniques used, and to obtain a value of Poisson's ratio.

Julius Miklowitz, USA

303. F. V. Warnock and J. A. Pope, "The change in mechanical properties of mild steel under repeated impact," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 26, pp. 33–44.

In the impact tests described the specimen is dropped a measured distance between two weights, the upper of which is the "head" and the lower of which is the "tup." The head strikes an anvil and the specimen absorbs not only the kinetic energy of the "tup" but also elastic energy of rebound of the "head" which was neglected in previous investigations using this apparatus. An extensometer designed to give the relation between extension and time is used. Assuming that the instantaneous strains are distributed uniformly over the gage length, an analysis is made of the extensometer data to determine maximum tensile stress and extension, and maximum compressive stress and deformation.

Stress extension diagrams for different energies of repeated impacts are compared with static tensile test and static repeated-loading data. In addition, comparisons are made between tests with equal energies but different velocities of impact. The variation of extension per impact during successive impacts is explained, and the effect of repeated impacts on the stress-strain diagram is indicated.

The effect of carbon content upon the ratio of impact energy of fracture to static energy of fracture is noted. The results are compared with those of other investigations, with corrections made when considered necessary. The authors emphasize the conclusion that while total extensions sustained under repeated impacts exceed those sustained during a tensile test, the ductility of a bar is reduced by repeated impacts.

Franklin H. Fowler, Jr., USA

Mechanics of Forming and Cutting Processes

(See also Rev. 280)

304. G. Sachs and L. J. Klingler, "The flow of metals through tools of circular contour," *J. appl. Mech.*, June 1947, vol. 14, pp. 88–98.

This paper is a theoretical analysis of fourteen cases of rolling wide strips and drawing and extruding of thin-wall tubes encountered in commercial metal-forming processes, considering various combinations of driving and driven rolls, roll diameter, longitudinal force and frictional restraint. The analyses are based on the assumptions of plane strain, assuming complete constraint against lateral spreading, for which Nadai [*J. appl. Mech.*, 1939, vol. 6, p. 56] gave the condition of plasticity that the sum of the roll pressure and longitudinal stress equals 1.15 times the flow stress for uniaxial tension.

General solutions for the roll pressures and stresses in the formed metal in terms of the flow stress are presented for the following cases: (1) symmetrical tools, no friction; (2) drawing and extruding, stationary dies, constant friction coefficient; (3) rolling without tension; (4) rolling, front and back tension; (5) pulling through undriven rolls; (6) rolling with one driven and one undriven roll; (7) forming with one roll and one stationary die; (8) general case of tools of different contour radii and constant friction coefficient; (9) tube drawing with stationary die and stationary mandrel; (10) tube drawing with roller dies and stationary mandrel; (11) tube rolling with stationary mandrel; (12) tube drawing, stationary dies, driven mandrel; (13) tube drawing, roller dies, driven mandrel; (14) tube rolling with moving, undriven mandrel; (15) tube forming with tapered mandrel; (16) forming with different friction coefficients at the two contact surfaces, one driven and one stationary tool. The results of many of these cases are shown by curves.

Henry A. Lepper, Jr., USA

305. A. Geleji, "Calculation of forces encountered in rolling processes and of required work compared with test data (Berechnung der beim Walzen auftretenden Kräfte und der Walzarbeit im Lichte der Versuchsergebnisse)," *Schweiz. Arch.*, Nov. 1947, vol. 13, pp. 336–344.

A simplified analysis of the rolling problem is presented which leads to the ready calculation of rolling forces and power requirements for rectangular bars. The analysis assumes a homogeneous material which follows the maximum shearing stress criterion for yielding, dry friction along the rolling surface and no lateral dimensional change during rolling. Further simplification is made by assuming the distribution of slip velocity between the bar and the roll, as well as an empirical expression for the longitudinal stress. With this stress and the assumed strength theory a stress is calculated which is considered to be the average rolling pressure from which the rolling force, torque and required horsepower are obtained.

The previously published experimental results of Puppe, Siebel, Emicke, and Lucas are compared with the calculated results obtained from this analysis. Aluminum bars rolled cold and steel rolled hot are studied in relation to the torque, rolling force, and rolling pressure for various reductions, rolling velocities and bar thicknesses. The analytical values agree well in most cases with those obtained experimentally. Louis F. Coffin, Jr., USA

Soil Mechanics; Seepage

306. Jean Mandel, "On small deformations of media possessing internal friction (Sur les petites déformations des milieux doués de frottement interne)," *C. R. Acad. Sci., Paris*, Oct. 20, 1947, vol. 225, pp. 666–667.

This short note gives equations, without derivations, representing the deformation (yield or consolidation) of dry soils in terms of the three principal stresses and moduli of elasticity. The soil is assumed to be an isotropic, homogeneous continuum. For saturated soils additional coefficients are suggested which can be substituted in the above equations. Three cases are discussed, covering elastic and rigid soils, as well as nonhomogeneous soils consisting of a rigid medium with nonrigid inclusions.

R. K. Bernhard, USA

307. N. K. Kalinin and P. J. Polubarinova-Kochina, "On unsteady motion of ground water with a free surface" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), Mar.–Apr. 1947, vol. 11, pp. 231–236.

To study the amount of sinking y_A of the horizontal water table due to the action of a horizontal underdrain, and the discharge q

of the latter, a complex velocity potential ω and a complex variable w proportional to the complex pressure are introduced and interconnected by a function z. The region occupied by water is mapped conformally on the region of an auxiliary complex variable ζ, making the water table coincide with the horizontal real axis of the map.

The boundary conditions at the free water surface are expressed by using the imaginary parts of some functions of ω, w, ζ, z and the time t. The values of the sinking y_A and the discharge q are expressed as power series of t. For times close to $t = 0$ the discharge q may be larger or smaller than the initial discharge q_0.

D. Krynine, USA

308. P. J. Polubarinova-Kochina, "Hydraulic theory of wells in a stratified medium" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, 1947, vol. 11, no. 3, pp. 357–362.

The paper refers to earlier work of A. N. Miatiev published in the 1946 Proceedings of the Turkmen Branch of the Academy of Sciences of the USSR and derives Miatiev's equations by a somewhat different procedure. Miatiev studied the case of flow through a pervious layer sandwiched between two less pervious layers. It was shown that even a slight permeability of these boundary layers may have a considerable effect on the discharge through the central pervious layer when vertical infiltration into it occurs over a considerable area. This may happen in oil-bearing strata. Reference is also made to Dutch studies of related problems reported by Krul and Liefrinek in 1946 in Holland.

The main body of the present paper is concerned with the development of Miatiev's method by the derivation of equations for the case of simultaneous axially symmetrical horizontal seepage toward a well through three pervious layers sandwiched between five less pervious layers, with pervious strata under a constant head at the extreme upper and lower boundaries. A general solution is given. The analysis of a special solution for the case when only the central of the three pervious layers is tapped by the well shows that under such conditions Miatiev's original formula for seepage through one pervious layer only can be used as a satisfactory approximation.

The paper ends with a short summaryin English, but the following misprints in this summary require correction. The symbol "ρ" for the radius of the well has slipped from the explanation of terms referring to formula 4.5 The formulas 2.7, 4.7 and 4.8 do not appear in the Russian text and references in the English summary to these formulas should be corrected to formulas 2.6, 3.5 and 4.3 in the order given. The word "power" of a layer is a literal translation of the Russian term and is intended to mean "thickness" of a layer. Gregory P. Tschebotarioff, USA

Potential Flow of Incompressible Fluids

(See also Revs. 320, 321, 340, 357, 358, 362, 364)

309. L. I. Sedov, "On the theory of the unsteady motion of an airfoil," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1156*, July 1947, pp. 1–62 (transl. from a publication of the Central Aero-Hydrodyn. Inst., USSR).

The unsteady motion of a cylindrical body in a two-dimensional incompressible flow is analyzed by standard techniques. Only the cases of zero and constant circulation are treated, so that the results are of limited interest to the flutter analyst. A table of virtual mass coefficients for Joukovsky airfoils is given. The author's premise that the circulation must be constant if the pressure is to be single values is, in the reviewer's opinion, incorrect. Since no reference is made to such important papers in this field as those of von Kármán and Sears [*J. aero. Sci.*, 1938, vol. 5, p. 379], L. Schwarz [*Luftfahrtforschung*, 1940, vol. 17, p. 379], and H. G. Kussner [*NACA T. M. No. 979*, 1941], it would appear that the paper might have been written over ten years ago. John W. Miles, USA

310. D. Gilbarg, "On the flow patterns common to certain classes of plane fluid motions," *J. Math. Phys.*, Oct. 1947, vol. 26, pp. 137–142.

It is proved that (1) in all plane potential flows of an incompressible fluid with the same streamlines, either the velocities at corresponding points are proportional, or the speed is constant along each streamline; and (2) the only families of streamlines common to plane potential flows of both compressible and incompressible fluids are concentric circles, or radii.

W. G. Bickley, England

311. D. E. Dolidze, "Linear boundary problem for the unsteady motion of a viscous incompressible fluid" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Mar.–Apr. 1947, vol. 11, pp. 237–250.

The fluid in the problem occupies an internal or external region of a closed regular surface F. The linear equations of motion are

$$\nu \Delta v - \frac{\partial v}{\partial t} = \frac{1}{\rho} \operatorname{grad} p, \quad \operatorname{div} v = 0 \left(\Delta = \sum \frac{\partial^2}{\partial x_i^2} \right)$$

where x_i ($i = 1, 2, 3$) denotes the rectangular co-ordinates, v_i the velocity components, p the hydrodynamical pressure, ρ the density, ν the kinematic coefficient of viscosity.

The initial and boundary conditions are:

$$(v_i)_F = f_i \quad (t > 0); \quad (v_i)_{t=0} = 0$$

In the external region there are also conditions at infinity which must be satisfied.

The paper proves the uniqueness of the solution. Further, after reducing the initial condition to zero, the author proceeds to construct a system of fundamental solutions of the above equations, thereby reducing the problem to a system of linear integral equations. The solution of these integral equations exists in the arbitrary finite interval of time, and may be found by successive approximation.

A bibliography containing eight references is included.

D. Jacovleff, Belgium

312. G. F. Carrier, "On the conformal mapping of airfoils" (in English), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan.–Feb. 1947, vol. 11, pp. 65–68.

The article gives a formal solution for the incompressible potential flow through a lattice of arbitrary airfoils (turbine or compressor blades). The airfoils are assumed to lie near the lines $y = \pm 2n\pi$. The transformation $\dfrac{e^z + 1}{e^z - 1}$ maps the exterior of the airfoil lattice on the interior of a closed curve lying near the unit circle.

By supplementary transformations the problem is reduced to the standard problem of transforming an approximately circular curve to a truly circular one.

Robert Jones, USA

313. G. H. Peebles, "A method for calculating airfoil sections from specifications on the pressure distributions," *J. aero. Sci.*, Aug. 1947, vol. 14, pp. 451–456.

This paper contains a procedure whereby one may find that profile which corresponds to an approximately prescribed pressure

distribution at a given lift coefficient or a qualitatively prescribed pressure distribution over a range of angles of attack. The procedure uses analytic functions of a complex variable and is essentially a search for an appropriate mapping function.

"Compatibility conditions" relating to the closure of the profile are discussed in detail, and statements are made as to the time required to construct such profiles. A hint is given for a procedure for investigating profiles with leading edge suction ports.

George Carrier, USA

314. Ion Cârstoiu, "Circulation in a viscous incompressible fluid (De la circulation dans un fluide visqueux incompressible)," *C. R. Acad. Sci., Paris*, Feb. 24, 1947, vol. 224, pp. 534–535.

The author shows that in the case of a viscous incompressible fluid if a potential function $\varphi(x_1, x_2, x_3, t)$ defined by

$$\Delta u_i = \frac{\partial \varphi}{\partial x_i}, \quad i = 1, 2, 3$$

exists, where u_i is the velocity vector of the flow and Δ the Laplacian operator, the circulation along a curve L moving with the fluid defined by

$$C_L = \int_L u_i \, dx_i$$

does not vary with time. Y. H. Kuo, USA

315. Ion Cârstoiu, "On the possibility of irrotational movements of an incompressible viscous fluid (Sur la possibilité des mouvements irrotationnels d'un fluide visqueux incompressible)," *C. R. Acad. Sci., Paris*, Oct. 20, 1947, vol. 225, pp. 664–666.

The author demonstrates the impossibility of the existence of a velocity potential φ defined by $u_i = \frac{\partial \varphi}{\partial x_i}$ (u_i being the velocity vector of the flow) in the case of a viscous incompressible fluid. Expressions giving the variation with time of both vorticity and velocity of the flow were derived by the Lagrangian method.

Y. H. Kuo, USA

316. Henri Cabannes, "Study of the steady flow of a viscous incompressible fluid (Étude de l'écoulement permanent d'un fluide visqueux incompressible)," *C. R. Acad. Sci., Paris*, Mar. 10, 1947, vol. 224, pp. 711–713.

A theoretical study of some incompatible factors is presented, which attempts to explain certain apparent paradoxes. The claim is made that no solution exists for the exact equations of Navier for steady, viscous flow around a fixed body with uniform flow at infinity. R. C. Binder, USA

317. S. G. Nuzhin, "Calculation of potential flow of an incompressible fluid past an airfoil of arbitrary shape" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan. 1947, vol. 11, pp. 55–64.

The paper develops a method of successive approximations for determining analytical functions which map a circle on the exterior of airfoils of arbitrary shape. These functions are represented by Laurent series, with the origin at the center of the circle; the coefficients of the terms of the series and the radius of the circle are determined by successive approximations. The airfoil is defined on a complex plane, the real axis coinciding with the maximum chord.

In a first approximation, the airfoil is replaced by a plate at the same angle of attack, or by any other reasonable approximation whose mapping on a circle is known, and the corresponding Laurent mapping series is defined. Then, points of the actual airfoil having the same abscissas as those of the first approximation define corresponding ordinates. The ordinates thus defined are used to determine the points on the mapping circle which have the same polar angle, defining at the same time the radius of a new mapping circle and the abscissas of the actual points which are represented on the new circle. These points provide the basis for a second approximation to the given airfoil, and can be used in the same manner as the first approximation. Thus, a method of successive approximations is defined, whose convergence has been theoretically proved by the author.

From an example, the convergence appears to be extremely rapid, and one to three cycles of approximations are required to obtain approximations of the order of a few thousandths on the coefficients of the Laurent series. Lift and moment coefficients are determined as functions of these coefficients.

Leon Beskin, USA

318. Oleg Yadoff, "Flow calculations in the steady Poiseuille flow (Sur le calcul des débits dans les écoulements premanents à la Poiseuille)," *C. R. Acad. Sci., Paris*, Feb. 10, 1947, vol. 224, pp. 374–376.

An analytical study is presented of the viscous flow through a rectangular channel, and some previous work by M. J. Kravtchenko is extended. The basic relation is transformed into elliptical integrals. This makes it possible to use existing tables for the evaluation of limiting cases, such as the flow between parallel plates and the flow through a square channel.

R. C. Binder, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 318, 329, 335, 352, 362, 363)

319. Neal Tetervin, "Boundary layer momentum equations for three-dimensional flow," *Nat. Advis. Comm. for Aero. Tech. Note No. 1479*, Oct. 1947, pp. 1–27.

The author develops formally the boundary layer equations for three-dimensional flow near a flat or slightly curved surface, both the density and viscosity being allowed to vary with position. Momentum integral equations analogous to those originally derived by von Kármán for two-dimensional flow are then obtained.

The remainder of the paper consists of checks that these equations are algebraically correct. It is shown that they can be reduced: (1) to similar equations for constant density and viscosity obtained recently by L. Prandtl ["On boundary layers in three-dimensional flow," *British MAP Volkenrode, Reps. and Trans.*, no. 64, 1946]; (2) to von Kármán's momentum equations for two-dimensional flow; and (3) to momentum equations given by A. Kehl ["Investigations on convergent and divergent turbulent boundary layers" (in German), *Ing.-Archiv*, 1943, vol. 13, pp. 293–329] for radial flow in two-dimensional converging and diverging channels. G. K. Batchelor, England

320. John R. Weske, "Experimental investigation of velocity distributions downstream of single duct bends," *Nat. Advis. Comm. for Aero. Tech. Note No. 1471*, Jan. 1948, pp. 1–39.

This paper describes measurements of velocity distributions in curved ducts, which were undertaken to provide engineering data for designers, as well as to determine the basic factors affecting the velocity distribution. Measurements were made at Reynolds numbers from 0.2 to 0.6 million at the outlet of the duct elbows, using round, elliptical, square, and rectangular cross sections. Three distinct regions of flow in curved ducts were investigated by different techniques of measurement. Special investigations included measurement of the decay of

velocity patterns produced by the curved ducts and of the effect of asymmetrical upstream velocity distributions.

The results indicate that cross-sectional shape and aspect ratio are minor factors compared to the radius ratio, in affecting the velocity pattern. Variation of Reynolds number within the range of the tests had no appreciable effect on the velocity distribution. The length of straight-approach duct relative to the curved duct, and poor matching of the duct joints both influence the upstream velocity pattern and have a noticeable though slight effect upon the velocity distribution in the curved ducts.

P. C. Chu, USA

321. Gabriel Viguier, "Prandtl boundary layer with important velocity gradients (La couche-limite de Prandtl avec d'importants gradients de vitesses)," *C. R. Acad. Sci., Paris,* July 7, 1947, vol. 225, pp. 45–46.

In a previous note ["Boundary layer equations for the case of large velocity gradients (Les équations de la couche limite dans le cas de gradients de vitesse élevés)," *C. R. Acad. Sci., Paris,* Mar. 10, 1947, vol. 224, p. 713] the author derived an equation for flow in a laminar boundary layer, suitable for the case of assumption that the stresses are linear functions of the rates of strain.

In the present note this theory is applied to a flow representing the type in the neighborhood of a stagnation point. A differential equation is derived for the boundary layer flow, which is a generalization of that of Falkner and Skan for the classical case [S. Goldstein, "Modern Developments in Fluid Dynamics," vol. 1, p. 140].

Louis Landweber, USA

322. R. G. Huebscher, "Friction equivalents for round, square and rectangular ducts," *Heat. Pip. Air Condit.,* Dec. 1947, pp. 127–135.

Air-friction tests in galvanized-metal ducts of circular, square, and 8 to 1 rectangular section, at Reynolds numbers (based on hydraulic diameter) from about 20,000 to 600,000, indicate that pressure-drop coefficients expressed in terms of hydraulic diameter are independent of the shape of section.

The pressure-drop coefficient may be defined as $\lambda = (D\Delta p)/(\frac{1}{2}\rho V^2 L)$, where the hydraulic diameter $D = 4A/P$, A and P are the section area and perimeter, Δp is the pressure drop over a length L, ρ is density, and V is mean velocity of flow. The variation of λ with Reynolds number is found to be very nearly that for smooth pipe. A table giving the radii of circular ducts having the same pressure gradient and capacity as rectangular ducts is included.

To estimate the validity of extrapolating the experimental results to higher aspect ratios, the reviewer computed the pressure-drop coefficient for a rectangular duct of infinite aspect ratio, i.e., for flow between parallel walls, the diameter being twice the distance between the walls. Assuming the $1/7$th power law of velocity distribution, the pressure-drop coefficient was found to be 6 per cent greater than for a circular duct.

Louis Landweber, USA

323. G. K. Batchelor and A. A. Townsend, "Decay of vorticity in isotropic turbulence," *Proc. roy. Soc., London, Ser. A,* Sept. 9, 1947, vol. 190, pp. 534–550.

The equation for the rate of change of the mean-square component $\overline{\omega^2} = 5\overline{u^2}/\lambda^2$ of the vorticity is written as:

$$\frac{d\overline{\omega^2}}{dt} = \frac{7}{3\sqrt{5}}\,(\overline{\omega^2})^{3/2}\,S - \frac{14}{3\sqrt{5}}\,(\overline{\omega^2})^{3/2}\,\frac{G}{R_\lambda}$$

where the first term on the right is due to the extension of the vortex lines, and the second is due to the effect of viscosity. Three correlation derivatives, expressed as Taylor's diffusion-length parameter λ, the skewness S of the $\partial u/\partial x$ distribution, and G defined in terms of $\overline{\left(\frac{\partial^2 u}{\partial x^2}\right)^2}$, are determinable from electrical (time) differentiation of the fluctuating velocity, on the assumption that $\frac{\partial u}{\partial x} = -\frac{1}{U}\frac{\partial u}{\partial t}$.

Hot-wire measurements are reported in a wind tunnel behind grids for mesh distances (the ratio x/M of distance x from grid to mesh size M) up to 120, and Reynolds numbers (UM/ν) of 6×10^3 to 4×10^4. The authors find $U^2/\overline{u^2}$ to increase linearly with x/M, and λ^2 to increase linearly with x. The variation with velocity indicates a linear increase with decay time. The Reynolds number of turbulence $R_\lambda = \sqrt{\overline{u^2}}\,\lambda/\nu$ was approximately constant during decay, while S was nearly constant at 0.39, and G was a linear function of R_λ. The values of G were in good agreement with those from the vorticity equation, with $S = 0.39$.

The results are interpreted to indicate that the two contributions to $d\overline{\omega^2}/dt$ are both proportional to $(\overline{\omega^2})^{3/2}$ and that the vorticity balance is essentially the same during decay. The viscosity contribution is $(1 + 22/R_\lambda)$ times the other. It is deduced that the shapes of the double and triple velocity correlations are preserved during decay, and with increasing Reynolds number the slope of the double velocity correlation curve just outside the parabolic region becomes smaller and the curve becomes more cusplike.

J. M. Robertson, USA

324. G. Moretti, "The theory of turbulent transport (La teoria del trasporto turbolento)," *L'Aerotecnica,* Feb. 1947, vol. 27, pp. 44–50.

In this paper the author presents a short critical review of the well-known "turbulent transport" theories of L. Prandtl, G. I. Taylor, and O. D. Mattioli. Since their only result is to give some estimate of Boussinesq's coefficient ϵ

$$-\overline{\rho u'v'} = \epsilon\,\frac{\partial U}{\partial y}$$

(where U is the velocity of the mean motion, u' and v' are fluctuations of the turbulent velocity, and $-\overline{\rho u'v'}$ is Reynolds stress), he concludes that it is better to avoid any hypothesis referring to some kind of turbulent transport.

He assumes simply that, since ϵ depends only on the mean motion, the Reynolds stress might be expressed in terms of U, $\partial U/\partial y$, $\partial^2 U/\partial y^2$, ...

$$\overline{u'v'} = \frac{\partial U}{\partial y}\,f\left(U, \frac{\partial U}{\partial y}, \frac{\partial^2 U}{\partial y^2}, \ldots\right)$$

Expanding the arbitrary function f he gets

$$\overline{u'v'} = \frac{\partial U}{\partial y}\left(a_1 U' + a_2 U + a_3{}^2\,\frac{\partial U}{\partial y} + a_4{}^3\,\frac{\partial^2 U}{\partial y^2} + \ldots\right)$$

where U' is a constant velocity and a_1, a_2, a_3, a_4 are lengths. The particular assumption $a_1 = a_2 = a_4 = 0$ leads to the Prandtl-Taylor formula, and the assumption $a_2 = a_3 = a_4 = 0$ leads to Mattioli's formula.

J. Kampé de Fériet, France

325. G. Moretti, "Plane turbulent wakes (Scie piane turbolente)," *L'Aerotecnica,* June 1947, vol. 27, pp. 210–221.

The author deals with the two-dimensional turbulent wake of a cylindrical body, the general assumptions being the same as in the well-known paper of H. Schlichting, [Ueber das ebene windschattenproblem, *Ing.-Archiv,* 1930, p. 533]. However, in accordance with his previous paper [*L'Aerotecnica,* Feb. 1947, vol. 27,

pp. 44–50] he does not make use of any transport theory. His main purpose is the computation of the lengths a_1, a_2, a_3, a_4 required in the previous paper. Assuming only that a_1, a_2, a_3, a_4 are expandable in series of powers of $\sqrt{a/x}$ (where a is the product of the coefficient of resistance of the body by its diameter) he obtains a general solution of the problem yielding as particular cases the solutions of Prandtl and Mattioli.

Some experimental data of Schlichting seem to agree better with the pressure distribution in the wake computed by the author, than with the curves computed by the Prandtl and Mattioli methods. J. Kampé de Fériet, France

326. V. G. Nevzgliadoff, "Theory of turbulent movement of compressible fluids" (in Russian), *Notes Acad. Sci. USSR (Doklady, Ak. Nauk, SSSR)*, 1947, vol. 58, no. 4, pp. 547–550.

This is one of a series of papers which have been published by the same author, who considers the problem of establishing a system of equations describing turbulent motion. The author first sets up a system of equations which generalize the Reynolds equations for compressible flow by the introduction of averaged quantities. The limits of applicability of these equations are then examined.

Next, the variables appearing in the six fundamental equations are separated into two groups. The first group is composed of six "fundamental" variables, while the second includes all the others—the "reducible" variables. The relations between the latter and the fundamental variables have been established by the author in a former paper [*J. Phys. Acad. Sci. USSR*, 1945, vol. 9, p. 235]. He gives to these relations the name of "dynamical equations of state." In the first approximation which has been studied by the author in another former paper [*Notes Acad. Sci. USSR, (Doklady, Ak. Nauk, SSSR)*, 1947, vol. 55, p. 107] the values of the coefficients occurring in these relations are assumed to be determined from experimental data.

By combination of these "equations of state" with the fundamental equations, the author obtains a system of six equations with six variables. He also determines the limits.

The author observes that the theory may be applied to aerodynamics of high-speed flow, and also to problems relating to the phenomena of exchange of heat by turbulent flow.

D. Jacovleff, Belgium

327. Gabriel Viguier, "Dissipation function in an isotropic turbulent flow (La fonction de dissipation en écoulement turbulent isotrope)," *C. R. Acad. Sci., Paris*, Dec. 22, 1947, vol. 225, pp. 1277–1278.

On the assumption that viscous stresses in a fluid are not proportional to the velocities of deformation, the author introduces, for the case of incompressible fluids, two new coefficients of viscosity, the dimensions of which are $ML^{-1}T$. To these coefficients correspond supplementary terms in the Navier-Stokes equations, together with supplementary terms in the dissipation function. The author also gives the revised form of the dissipation function in an isotropic turbulent flow. Leon Beskin, USA

328. W. S. Coleman, "Analysis of the turbulent boundary layer for adverse pressure gradients involving separation," *Quart. appl. Math.*, July 1947, vol. 5, pp. 182–216.

The author attempts to refine and extend the methods of von Doenhoff and Tetervin and of Garner for analyzing the development of turbulent boundary layers. He divides the boundary layer into a laminar sublayer and a turbulent layer, and applies the Prandtl momentum-transfer theory and von Kármán's similarity relation to the turbulent layer. Hence he regards this as a theoretical rather than an empirical treatment and states "it has not been possible to present . . . . . a unified theory."

All the methods characterize the boundary layer at a given position by its momentum thickness ϑ and shape parameter H (ratio of displacement thickness to momentum thickness). The basic independent variables are the Reynolds number R_δ based on the boundary-layer thickness δ and speed q_1 just outside the boundary layer, and ω_δ, the ratio to the surface friction τ_o of the product of δ by the pressure gradient $\partial p/\partial s$.

All the methods yield three equations. The momentum equation:

$$\frac{d\vartheta}{ds} = \frac{\tau_o}{\rho q_1^2}\left[1 + \frac{\vartheta}{\tau_o}\frac{\partial p}{\partial s}(H+2)\right]$$

is common to all. The second equation relates τ_o to R_δ and ω_δ. Von Doenhoff and Tetervin used the Squire-Young formula which neglects the effect of ω_δ, Garner used the Falkner formula and considers ω_δ, while the author derives and uses a relation obtained from his analysis of the velocity distribution. Von Doenhoff and Tetervin and Garner used differing empirical relations for a third equation relating $\vartheta\dfrac{dH}{ds}$ to ω_δ and R_δ. The author derives the third equation by applying the continuity requirement to a small section of the boundary layer of length ds, but in so doing introduces a new variable θ_1 (slope of outer edge of the boundary layer) which must be evaluated from the theoretical analysis.

In the theoretical analysis the shear stress distribution is assumed as a power series in the distance from the surface. The coefficients, which are functions of ω_δ, are determined from boundary conditions. A different approximation is used in determining θ_1 (which depends greatly on the laminar sublayer) from that used in determining τ_o and the relation of δ to ϑ. It is difficult to assess the adequacy of the approximations made. The results agree more closely with Garner than with von Doenhoff and Tetervin. Hugh L. Dryden, USA

Compressible Flow, Gas Dynamics

(See also Revs. 310, 326, 346, 350, 352, 353, 361, 381, 385)

329. Jean Fabri, "Boundary layer in supersonic flow (Couche limite en écoulement supersonique)," *C. R. Acad. Sci., Paris*, July 7, 1947, vol. 225, pp. 42–44.

The equation of conservation of mass for supersonic flow in the boundary layer of a two-dimensional profile is applied to derive a differential equation giving the variation along the profile of the velocity outside the boundary layer.

An application of the theory to the case of a flat plate, before separation, yields a relation which agrees well with the forms of previously published experimental curves.

Louis Landweber, USA

330. S. R. Brinkley, Jr., and J. G. Kirkwood, "Theory of the propagation of shock waves," *Phys. Rev.*, May 1, 1947, vol. 71, pp. 606–611.

This analysis extends the work of G. I. Taylor [*Proc. roy. Soc., London, Ser. A*, 1946, vol. 186, p. 278] who applied the Rankine-Hugoniot equations to obtain a numerical solution for the spherical shock flow in a perfect gas.

The authors apply Hugoniot's conditions for a fluid with an arbitrary equation of state, and assume that the nonacoustical decay of the shock flow is directly related to the varying entropy increment continually dissipated in the flow, and that the rate of the varying energy propagation can be approximated by an ex-

ponential function of the time. The equations of hydrodynamics are introduced in a hybrid form, using Euler's equation of continuity, and with the Lagrange co-ordinates of radial distance and time as the independent variables. This results in a pair of ordinary differential equations for either cylindrical or spherical shock waves. These equations can be integrated numerically to provide values for the variation of the peak pressure and shock-wave energy with distance from the source.

The reviewer adds that when applied near an explosive source the high temperatures encountered undoubtedly make Taylor's assumption of a perfect gas a crude approximation, and it would be better to use relations for an arbitrary equation of state. However, the temperature gradients are also extremely large near an explosive source, and therefore neither the Rankine nor Hugoniot relations are valid. Only a comparison with experimental data could prove whether there has been any appreciable improvement over Taylor's solution. E. V. Laitone, USA

331. S. R. Brinkley, Jr., and John G. Kirkwood, "Theory of the propagation of shock waves from infinite cylinders of explosive," *Phys. Rev.*, Dec. 1, 1947, vol. 72, pp. 1109–1113.

As the authors state, the assumption of energy dissipation at a single shock, formulated in their earlier paper ["Theory of the propagation of shock waves," *Phys. Rev.*, May 1, 1947, vol. 71, pp. 606–611], is employed in setting up a pair of ordinary differential equations for peak pressure and shock-wave energy as functions of radial distance from the source, for the case of a shock wave which is produced by an infinite cylinder of explosive along which a stationary detonation wave is traveling with finite velocity. The profile of the wave is determined by means of an auxiliary integration.

The theory takes proper account of the finite entropy increment of the fluid produced by the passage of the shock, and permits the use of the exact Hugoniot curve of the fluid in the numerical integration of the basic equations. C. A. Meyer, USA

332. A. H. Shapiro and W. R. Hawthorne, "The mechanics and thermodynamics of steady one-dimensional gas flow," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 317–336.

A one-dimensional analysis for flow without shocks is presented, which takes into account the simultaneous effects of area change, wall friction, drag of internal bodies, external heat exchange, chemical reaction, change of phase, injection of gases, and changes in molecular weight and specific heat. Examples are given of several simple types of flow, involving changes due to only one of the above effects. In addition, several examples of flow with combined effects are given, including: (*a*) simultaneous friction and area change; (*b*) simultaneous friction and heat transfer; and (*c*) simultaneous liquid injection and evaporation.

A one-dimensional analysis for flow through a discontinuity is presented, allowing for energy, shock, drag, and gas-injection effects, and for changes in gas properties. This analysis is applicable to such processes as: (1) the adiabatic normal shock; (2) combustion; (3) moisture condensation shocks; and (4) steady explosion waves. Arthur Kantrowitz, USA

333. E. Arthur Bonney, "Aerodynamic characteristics of rectangular wings at supersonic speeds," *J. aero. Sci.*, Feb. 1947, vol. 14, pp. 110–116.

Lift, drag and moment coefficients and center of pressure are derived for a rectangular wing in supersonic flow. The "Busemann second order approximation" is used to obtain pressure distributions for a two-dimensional airfoil and these are then modified at the wing tips, in the regions and to the extent indicated by linear conical-flow theory. The author concludes that: (1) Contrary to subsonic theory, the drag due to lift at constant angle of attack decreases with decreasing aspect ratio; (2) the center of pressure shifts forward as the thickness ratio increases; (3) there is no simple pattern for variation of the coefficients with Mach number; (4) airfoils with the same cross-sectional area have identical centers of pressure; (5) form drag varies as the thickness ratio squared for a given cross-sectional shape; (6) symmetrical-cross-section airfoils with maximum thickness at mid-chord have the least drag for a given thickness ratio or allowable stress.

In the reviewer's opinion, it has been shown [H. R. Ivey, *J. aero. Sci.*, July 1947, p. 432] that, because of an implicit assumption of symmetry in the handling of the second order terms, conclusion (4) is true only for airfoils symmetrical about the chord line, (5) is exact only for airfoils symmetrical about the chord line and the mid-chord, and (6) is incorrect. It would follow that the expressions for the coefficients apply only to airfoil cross sections with the proper symmetry. A. F. Donovan, USA

334. A. H. Taub, "Refraction of plane shock waves," *Phys. Rev.*, July 1, 1947, vol. 72, pp. 51–60.

The problem investigated is the flow resulting from the incidence of a plane shock wave on an interface between two gases of different densities and different ratios of specific heats. A three-shock configuration is assumed to result, involving the incident shock, a reflected shock, and a transmitted shock. The flow conditions are assumed to be influenced only by shocks which follow the Rankine-Hugoniot relations. These, together with the boundary conditions, give a rational polynomial of the twelfth degree, the roots of which determine the position and strength of the reflected and transmitted shock as functions of the strength, angle of incidence of the incident shock, and three parameters characterizing the pair of gases involved.

The author studies the solutions of the equation as multiple branched functions of the five parameters. It is shown that one branch behaves similarly to the acoustic case, and the author suggests that this branch is the only physically realizable one. Relations determining the ranges of parameters where real, physically realizable solutions may exist, show that the three-shock configuration is impossible for angles of incidence corresponding to the angle of total reflection.

Hints are also given as to the existence of solutions where the reflected shock is replaced by a Prandtl-Meyer expansion, and of solutions analogous to the Mach reflection. H. S. Tsien, USA

335. J. Ackeret, F. Feldmann, and N. Rott, "Investigations of compression shocks and boundary layers in gases moving at high speed," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1113*, Jan. 1947, pp. 1–34 (transl. from *Inst. f. Aerodyn. T. H. Zurich.*, publ. no. 10).

This report deals with the interaction of shock and boundary layers, which was investigated experimentally for a limited supersonic region such as occurs on an airfoil. A combination of schlieren photography and static total pressure measurements was used.

Laminar boundary layers were produced on a curved plate in a tunnel with adjustable curved walls. Use was made of the turbulent boundary layer on the tunnel wall, as well as the turbulent boundary layer produced on the plate by turbulence wires. Reynolds and Mach numbers could be varied independently.

The phenomena observed differed according as the boundary layer was initially laminar or turbulent. When the boundary layer was initially laminar and the greatest Mach number was only slightly greater than unity, a series of weak shocks occurred. As the Mach number increased, these decreased in number and increased in intensity. The normal phenomenon with boundary

layers initially laminar was a strong "main" shock preceded by an "oblique" shock, graphically described by the term "λ-shock." Behind the oblique shock the displacement thickness increased rapidly, at a rate corresponding to the calculated change of direction of the flow. Behind the main shock the layer was turbulent.

With boundary layers which were initially turbulent only the main shock appeared.

In all cases there was a considerable increase in the (displacement) thickness of the boundary layer, and this was greater for laminar than for turbulent layers. At the wall itself the pressure changes occurred gradually. W. G. Bickley, England

336. Stefan Bergman, "On supersonic and partially supersonic flows," *Nat. Advis. Comm. for Aero. Tech. Note. No. 1096*, Dec. 1946, pp. 1–85.

The author here brings together and extends the results of his previous theoretical studies relating to solutions of the hodograph equations for two-dimensional frictionless flow of a gas, wherein the restrictive assumptions of Chaplygin or of von Kármán and Tsien regarding the pressure - density relation are dropped.

For subsonic flows, proof is given of a theorem which adds considerably to the utility of the solution previously given by the author, involving the imaginary part of a function of a complex variable. Tables are given to facilitate calculation of coefficients in the series expression of this function.

For supersonic flows, proof is given of a certain key theorem which underlies the solution given by the author. A numerical example is carried out in this section of the paper, including final reduction to the physical plane x, y. In the final section a method is developed for extending a subsonic flow defined in a portion of the plane into a larger domain. In some instances this process will lead to partially supersonic flows.

The methods presented would not appear to lend themselves, directly, to the solution of problems involving prescribed boundary conditions. Stewart Way, USA

337. W. Hantzsche and H. Wendt, "Cones in supersonic flow," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1157*, Aug. 1947, pp. 1–27 (transl. from *Jb. dtsch. Luftfahrtforsch.*, 1942, p. I 80).

A graphical method for determining the supersonic flow over a cone was given by A. Busemann [*Z. angew. Math. Mech.*, 1929, vol. 9, p. 496]. The present paper gives the results of this procedure for various Mach numbers and cone angles so that a good survey of all possible conical-flow patterns is obtained. Analytical considerations are used where the accuracy of the graphical method is unsatisfactory. A representation of the shock-polar and the conical-flow pattern is derived for the limiting case in which the Mach number tends to unity. G. Guderley, USA

338. L. Viaud and A. Benoit, "On pressure measurements in a fluid flow with shock waves (Sur la mesure des pressions dans un ecoulement fluid avec ondes de choc)," *C. R. Acad. Sci., Paris*, July 16, 1947, vol. 225, pp. 96–98.

This reports one set of tests made through a shock wave which was stationary at the entrance of a nozzle. A Pitot tube was placed along the axis of the nozzle and moved fore and aft of the shock. Test results show large deviations from the theoretical curve (without Pitot) which is given. Leon Beskin, USA

339. B. Göthert and K. H. Kawalki, "The calculation of compressible flows with local regions of supersonic velocity," *Nat. Advis. Comm. for Aero. Memo. No. 1114*, March 1947, pp. 1–13 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1794, 1943).

This paper discusses an approximate method for calculating compressible flows about profiles with local regions of supersonic velocity. According to this method, the subsonic portion of the flow field is computed by the Prandtl-Glauert transformation up to points where the perturbation velocities are high enough so that the linearized approximation gives appreciable errors. From these points, the flow field is extended to the "sonic boundary" by numerical calculation using the exact equations in difference form.

The supersonic portion of the flow is determined by the method of characteristics, with the requirement that the contour of the body must be a streamline of the supersonic flow. To satisfy this boundary condition, the subsonic flow field must be adjusted by an appropriate dipole or source-sink distribution and the calculation repeated.

Examination of the characteristics net shows whether or not the transition from supersonic to subsonic velocities can be isentropic in a particular case. Qualitative results are given for flow around a particular slender profile, but no calculations are shown in the paper. E. D. Kane, USA

340. Max A. Heaslet, "Compressible potential flow with circulation about a circular cylinder," *Nat. Advis. Comm. for Aero. Rep. No. 780*, 1944, pp. 1–9.

An extension of the Rayleigh-Janzen method is presented giving the numerical second order solution (involving M^4) for the two-dimensional subsonic flow about a circular cylinder, with and without circulation. The results are compared with the Prandtl-Glauert and von Kármán-Tsien approximations. As would be expected, the Prandtl-Glauert approximation greatly underestimates the effect of compressibility when the pressure coefficient approaches — 2. The von Kármán-Tsien expression overestimates the compressibility effect, but still provides a more reasonable approximation.

The reviewer believes the surface pressures presented are in error, since the pressure relation must also be expanded in powers of M^2 to be consistent with the Rayleigh-Janzen development. The pressure coefficient should be written

$$P = \frac{2}{\gamma M^2}\left\{\left[1+\frac{\gamma-1}{2}M^2\left(1-\frac{V^2}{U^2}\right)\right]^{\gamma/\gamma-1}-1\right\}$$
$$= \frac{2}{\gamma M^2}\left\{\frac{\gamma M^2}{2}\left(1-\frac{V^2}{U^2}\right)+\frac{\gamma M^4}{8}\left(1-\frac{V^2}{U^2}\right)^2+\frac{\gamma(2-\gamma)}{48}M^6\left(1-\frac{V^2}{U^2}\right)^3---\right\}$$

The first two terms may be retained for the second order approximation, which decreases the values presented only slightly. However only the first term may be retained for the M^2 approximation, increasing the presented values of P by as much as 10 per cent, and eliminating the decreasing divergence which has no physical or theoretical basis. E. V. Laitone, USA

341. E. V. Laitone, "The linearized subsonic and supersonic flow about inclined slender bodies of revolution," *J. aero. Sci.*, Nov. 1947, vol. 14, pp. 631–642.

The author analyzes the order of approximation in the various known solutions for velocity potential of flows around slender bodies of revolution. The method of expanding in series the distance r from the body axis is consistently applied to: (*a*) axial flow, inclined flow, and pitching of the body at supersonic speeds; (*b*) axial flow and inclined flow at subsonic speeds.

The author shows that in the supersonic case certain terms presumably of higher order (the presence of which should be imma-

terial) tend to exceed terms of lower order in r because of the variation of their coefficients with Mach number. Such considerations lead him to the conclusion that the standard solutions of von Kármán-Moore and Tsien misrepresent the Mach number effect. Several of the statements of the author may well remain controversial. M. V. Morkovin, USA

342. K. P. Staniukovich, "The flow of detonation products in the case of an 'inclined' detonation wave" (in English), *C. R. Acad. Sci. URSS*, Feb. 10, 1947, vol. 55, no. 4, pp. 311–314.

The plane front of a detonation wave is assumed to move along the plane boundary of a charge, the angle α between these two planes remaining constant. The author apparently assumes a charge burning off at the same rate as the wave proceeds. In a system moving with the wave, the flow of the detonation products is described as a Prandtl-Meyer flow into vacuum around the instantaneous edge of the charge. This idea is carried out under the assumption that $c_i = Dn/(n+1)$, where c_i is the initial velocity of sound, D is apparently the detonation velocity, and $n = c_p/c_v$.

Curves showing the angular distribution of the energy of the detonation products are presented for the cases $\alpha = 45°, 90°$ together with $n = 1.4, 3.0$. Each of them shows a sharp peak, at a certain angle of back-scattering which depends on α and n. Good agreement is claimed with experiments which were made especially for the purpose. The paper is difficult to read because the notations are not sufficiently explained. G. Kuerti, USA

343. I. E. Garrick and S. I. Rubinow, "Theoretical study of air forces on an oscillating or steady thin wing in a supersonic main stream," *Nat. Advis. Comm. for Aero. Tech. Note No. 1383*, July 1947, pp. 1–44.

Methods for determining the forces acting on nonstationary or oscillating wings are of great interest in the fields of aircraft instability and flutter. The two-dimensional supersonic flow case (wing of infinite aspect ratio) has been solved by a number of investigators. The present paper extends the theoretical development to the three-dimensional case of finite wings of various plan forms. The treatment is based on the linearized theory, and is therefore applicable to nearly flat thin surfaces at small angles of attack in a nonviscous stream having constant supersonic velocity.

The fundamental solution of the wave equation corresponding to a moving spherical source is first discussed. The action of the wing is then represented by source and sink distributions, and integration of the fundamental solution for the source provides solutions for the velocity potential. The solutions obtained apply principally to the pure supersonic flow areas of the wing which do not include regions inside the Mach cone springing from the wing tips. Analysis of tip regions and sweptback or triangular wings lying within the Mach cone from the vertex requires more elaborate methods than used here.

A number of examples are given illustrating the use of the method in both harmonically oscillating and steady cases. These include the rectangular wing, constant-chord sweptback wing, and triangular wing. John V. Becker, USA

344. I. I. Eterman, "Distribution of pressure over the surface of a body of revolution in a gas flow of high subsonic velocity" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, May–June, 1947, vol. 11, pp. 363–370.

This paper represents a development of an approximate method of calculating compressible flows past a body of revolution, which was originated in 1940 by S. A. Christianovich.

This method makes use of an auxiliary plane, and the present paper gives a procedure for determining, in an approximate manner, the transformed meridian section, using the property that ellipses are transformed into ellipses with a great degree of accuracy. A procedure for integrating Christianovich's equations is given, and the general procedure is developed for the cases when the body is an ellipsoid of revolution or has a more general shape. A comparison with experimental results is given for the incompressible case. Leon Beskin, USA

Aerodynamics of Flight; Wind Resistance

(See also Revs. 317, 321, 333, 343, 344, 353, 354, 357, 358, 371)

345. L. G. Whitehead, "Low drag airfoils," *J. roy. aero. Soc.*, Jan. 1947, vol. 51, pp. 54–64.

This paper is a qualitative summary of the characteristics of the subject airfoils, with charts indicating the calculated influence of various design parameters. Some design considerations from both theoretical and application aspects are noted. No experimental data are included. J. M. Wild, USA

346. R. K. Page, "Performance calculation for jet-propelled aircraft," *J. roy. aero. Soc.*, May 1947, vol. 51, pp. 440–450.

Making the simplifying assumptions that net thrust is independent of flight speed and that specific fuel consumption is independent of engine speed for jet-propelled aircraft, the author treats analytically the conditions for maximum level speed, maximum rate of climb, absolute ceiling, range and endurance, and compares results with those for reciprocating engines. A graphical method is outlined for general performance computation in the compressibility region. M. V. Morkovin, USA

347. V. Golubev, "On the theory of wings of low aspect ratio" (in Russian), *Bull. Acad. Sci. URSS (Ser. techn. Sci.)*, 1947, no. 3, pp. 261–270.

The paper presents a method for computing the lift coefficient of wings of small aspect ratios. Using conventional theory, together with assumptions not wholly consistent with it, the author introduces a secondary circulation proportional to the square of the angle of attack, and obtains a correction to the conventional lift coefficient. There does not seem to be good correspondence between the theory and the test data given. For aspect ratios of 5 and 1 the corrections are far too great, while for an aspect ratio of 1/5, they are insufficient. Leon Beskin, USA

348. Milton M. Klein, "Pressure distributions and force tests of an NACA 65-210 airfoil section with a 50 per cent chord flap," *Nat. Advis. Comm. for Aero. Tech. Note No. 1167*, Jan. 1947, pp. 1–68.

This report gives the results of two-dimensional, low Mach number, high Reynolds number (3 to 9 million) tests in the NACA Langley Field low turbulence tunnel. Pressure distribution and force data are given for flap deflections of 0, 4, 7, and 10 deg. Theoretical calculations by Glauert's thin airfoil theory gave reasonably accurate correlation with test values for angles of zero-lift and pitching moment coefficients.

Conrad A. Lau, USA

349. N. G. Chetayev (editor), "N. E. Joukovsky" (in Russian and English), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Jan.–Feb., 1947, vol. 11, pp. 1–40.

This article is a biography of the Russian pioneer in applied mechanics, Nicholai Egorovich Joukovsky (1847–1921), pre-

sented in two sections, one in Russian and the other in English which does not appear to be a direct translation of the Russian. This review is based on the English section of the biography. There is also given what seems to be a complete bibliography of Joukovsky's published works covering the period from 1876 to 1920, followed by a list of papers written about him or based on his works during 1922–1939. The majority of Joukovsky's papers were written in Russian but there are occasional titles in German and French.

The biography begins with a brief outline of Joukovsky's family background and early education. This is followed by a critical review of his more important papers. The latter discussion is divided into three sections: (1) Hydrodynamics and hydraulics, (2) General mechanics, and (3) Aerodynamics.

American readers, who are perhaps most familiar with Joukovsky's contributions to airfoil theory, will undoubtedly be impressed by the breadth and variety of his work. In hydrodynamics he studied problems in ship hull resistance and stability, bearing lubrication, ground water flow, water hammer, and the behavior of vortices. His works in general mechanics included studies on the stability of motion of rigid bodies, on the vibration of turbine shafts, locomotives and automobiles, and contributions in the field of astronomy.

His many works in aerodynamics included the classical papers in 1910–1911 which formed a part of the foundation of modern subsonic airfoil theory. He was one of the first wind-tunnel experimenters and had a full appreciation of such difficulties as jet boundary effects. His work in compressible fluids paved the way for the more extensive studies in gas dynamics by one of his students, Chaplygin.

Joukovsky was truly one of the Titans in the field of applied mechanics for which his period was so notable.

M. J. Thompson, USA

350. A. Lippisch and W. Beuschausen, "Pressure distribution measurements at high speed and oblique incidence of flow," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1115*, March 1947, pp. 1–11 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin-Adlershof*, Res. Rep. no. 1669).

This report presents the results of pressure distribution measurements on a 9 per cent thick symmetrical airfoil, made in the high-speed tunnel of the Institute for Gas Dynamics at LFA-Braunschweig. The model was mounted so as to obtain essentially two-dimensional flow conditions. Tests were run at angles of yaw (sweepback) of 0, 20, and 40 deg; at angles of attack from 0 to 3 deg; and at Mach numbers ranging from 0.44 to 0.96.

For comparison with the measurements, the theoretical pressure distributions were determined exactly for the case of incompressible flow and then by using Prandtl's rule were modified to correspond to the test Mach numbers. For the wing at zero yaw the agreement between theoretical and measured pressures was good except at the leading edge of the model. For the wing at 20 and 40-deg yaw angles, the variation of normal force with Mach number at constant angle of attack showed a smaller increase than would be predicted from the Prandtl rule. The results confirm the beneficial effect of sweepback for wings at high Mach numbers. Dana Young, USA

351. R. W. Holtzclaw and Jules B. Dods, Jr., "Wind-tunnel investigation of drooped ailerons on a 16 per cent thick low-drag airfoil," *Nat. Advis. Comm. for Aero. Tech. Note No. 1386*, Aug. 1947, pp. 1–69.

The practicability of using ailerons as high-lift devices was investigated in the wind tunnel. In practice the ailerons are drooped when the flaps are lowered, and the usual differential deflection of the ailerons is employed for lateral control.

Wind-tunnel tests were conducted at a Mach number of 0.19 and a Reynolds number of approximately 5,100,000 on two ailerons of different shape on a low-drag airfoil. The effect of aileron position and deflection on the airfoil lift coefficient, the variation of the hinge moment coefficient of the aileron and the effect on the airfoil section drag and pitching moment characteristics were studied. Calculations are made, based on these results, which show that drooped ailerons may perform satisfactorily on several types of airplanes. A. Wiley Sherwood, USA

352. B. Regenscheit, "Drag reduction by suction of the boundary layer separated behind shock-wave formation at high Mach numbers," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1168*, July 1947, pp. 1–18 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1424, 1941).

Wind-tunnel experiments were made of the high-velocity flow past wing models having sections formed by two circular arcs, with rounded nose. The drag was measured by a wake traverse, observations were made with the schlieren technique, the Mach number was varied, and the suction flow was also varied. Tests were made with different positions and arrangements of the suction slits.

The results showed that the drag could be reduced by a suction of the boundary layer. Such suction of the boundary layer changed the position and shape of the shock waves. The most favorable results were obtained with a suction slit located at 70 per cent of the wing chord. R. C. Binder, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 212, 234, 270, 309, 343)

353. S. V. Falkovich and M. D. Haskind, "Vibrations of a wing of finite span in a supersonic flow," (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), May 1947, vol. 11, pp. 371–376.

The paper gives a theoretical solution to the problem of a supersonic flow past an infinitely thin vibrating delta wing, under the assumptions of the conventional linearized-flow theory for non-steady motion. The wing deformations, defined by an arbitrary distribution of small slopes over a reference plane which replaces the actual surface, are developed in a Fourier series of the time.

The constant term and a harmonic are examined, corresponding respectively to an arbitrary initial shape and arbitrary deformations. For the constant term, and for each harmonic, the corresponding asymptotic states of flow (steady and periodic) are defined in terms of complex potentials. The latter are represented by multiple series defined in curvilinear co-ordinates, and containing Bessel and harmonic functions of the latter. The method is limited to symmetrical delta vings entirely inside their Mach cones. No application is given, and the effective numerical procedure appears to be quite involved. Leon Beskin, USA

354. Gabriel Horvay, "Rotor-blade flapping motion," *Quart. appl. Math.*, July 1947, vol. 5, pp. 149–167.

A method developed previously by the author for the solution of a class of Hill differential equations is applied to the solution of the differential equation of the flapping motion of helicopter rotor blades.

The stability of the flapping motion is established on the basis of the following assumptions: (*a*) The blades are attached to the rotor shaft by means of flapping hinges only, the distance of the hinges from the rotor axis being negligible; (*b*) the hinge axes are

perpendicular to the rotor shaft and to the respective blade span axes; (*c*) the blades are untwisted, and are rigid both in bending and in twist; (*d*) the chord and the lift coefficient remain constant along the blades; (*e*) the uniform motion of the helicopter is not affected by the oscillations of the blades; (*f*) only the velocity square lift forces are taken into account as acting aerodynamically on the blades; (*g*) air velocities in the direction of the blade span are neglected; (*h*) the field of induced velocities is assumed constant over the rotor disk, and interference effects between blade elements are neglected; (*i*) all angles (other than the angle of rotation) are small, and the angle of attack is always less than the stalling angle.

The principal practical result obtained is a calculation of the percentage decrease per revolution of the amplitude of a transient. This calculation indicates a curious behavior of the flapping transients for a certain value of the aerodynamic damping coefficient n. For this value of n the stability decreases as the advance ratio μ increases, and the natural frequency of the flapping motion is half the angular velocity of the blade over a wide range of μ's and n's. The calculation also shows the sluggishness of the blades in recovering their steady-state motion when the value of n is small. The conclusion is that blades with low aerodynamic damping coefficients are to be avoided.

The paper ends with a discussion of steady-state response, and the author concludes that for helicopters resonance excitation is as remote as is instability. Edward Saibel, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 202, 299, 312, 354, 372)

355. Eckert and Korbacher, "The flow through axial turbine stages of large radial blade length," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1118*, Apr. 1947, pp. 1–26 (transl. from *Dtsch. Luftf.*, Rep. no. 1750).

The flow through blading in axial-flow steam turbines is affected by the variation in pressure along the blade length. This condition with its resultant effect on efficiency becomes significant when the blades are relatively long.

Studies were made for untwisted blades with 30 and 45-deg exit angles. The variation in pressure distribution in conjunction with the centrifugal effect due to the whirl component of flow causes a radial inward flow in the untwisted stator blades and a radial outward flow in the untwisted rotor blades. A theoretical method is presented for determining the effect in both twisted and untwisted blading. The method of analysis is applicable to axial-flow compressors and gas turbines.

Tests were made on untwisted blades with 30 and 45-deg exit angles. These show that for a real gas subject to friction the results closely confirm the calculations. Curves, sketches and photographs are given to illustrate the results.

E. G. Allen, USA

356. J. F. M. Scholes, "Ducted fans: a nomogram method of analysis," *Austral. Counc. Aero.*, Feb. 1947, no. 32, pp. 1–22.

This paper is the sixth of a series on ducted axial-flow fans or multibladed propellers. It deals particularly with nomographic-chart methods for designing multibladed propellers such as are used in wind tunnels or low-pressure-ratio blowers. Such fans are usually designed to perform at a maximum peak efficiency for a given ratio of axial mean velocity to blade-tip speed. The nomographic method provides a means of reducing the design calculation work.

The procedure for reading the charts is described and an example of a calculation for a typical fan is given: (1) without straightening vanes, and (2) with a 7-blade straightener. An example is also given of the use of the charts on a set of contra-rotating fans. Curves are given for experimental and calculated performance coefficients. Good agreement between the experimental tests and the chart computation is evidently obtained. A two-section appendix presents detailed sample calculations in column form. Compressibility and blade interference limitations are imposed as factors in the choice of parameters to make the theory reliable.

To get the greatest value from a study of this paper the original references by G. N. Patterson in *Austral. Counc. Aero.*, nos. 7, 8, 9, 10, and 14 should also be consulted. Six 20 × 20-in. working nomograms may be obtained from the Austral. Counc. for Aero., Melbourne, Australia. Millard Dowell, USA

357. S. Katzoff, R. S. Finn, and J. C. Laurence, "Interference method for obtaining the potential flow past an arbitrary cascade of airfoils," *Nat. Advis. Comm. for Aero. Tech. Note No. 1252*, May 1947, pp. 1–55.

A practical computational method for determining the incompressible potential flow about airfoils in cascade is presented in this paper. The flow about an isolated airfoil (primary airfoil), having the desired shape and angle of attack, is first determined by standard methods. The result is then modified to take account of adjacent airfoils in the cascade (interfering airfoils) by the introduction of appropriate sources and vortices over their surfaces. The interference calculation is considerably simplified by a graph which has been prepared by Albert Betz for this purpose ["Diagrams for calculation of airfoil lattices," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1022*, 1942].

The singularities at the interfering airfoils are then adjusted by iteration to conform to the velocity distribution determined for the primary airfoil. It is shown that the iterative process need be carried out for only one angle of attack.

Robert T. Jones, USA

358. Leo Diesendruck, "Iterative interference methods in the design of thin cascade blades," *Nat. Advis. Comm. for Aero. Tech. Note No. 1254*, May 1947, pp. 1–29.

This paper is an application of the method of Katzoff, Finn, and Laurence [*Nat. Advis. Comm. for Aero. Tech. Note No. 1252*] to the determination of the camber shapes necessary to produce certain desired variations of the velocity through a cascade of very thin airfoils. Robert T. Jones, USA

359. L. J. Herrig and S. M. Bogdonoff, "Performance of an axial-flow-compressor rotor designed for a pitch-section lift coefficient of 1.20," *Nat. Advis. Comm. for Aero. Tech. Note No. 1388*, July 1947, pp. 1–18.

This describes low-speed tests of an axial-flow-compressor rotor with highly cambered blades. The test configuration did not include any stator blades, and the inlet velocity distribution was uniform and free from appreciable boundary layer. Under these idealized conditions a rotor efficiency of 96 per cent was obtained at a mean blade-lift coefficient of 1.2. The results indicated that even higher lift coefficients might be obtainable with high efficiency. It remains to be determined whether lift coefficients of this magnitude can be used efficiently in the presence of appreciable boundary layers and interference effects from adjacent blade rows. John V. Becker, USA

360. H. H. Anderson, "Centrifugal pumps—an alternative theory," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 27, pp. 57–63.

Performance data (head versus inflow) of several low and medium pressure centrifugal pumps are compared with head figures calculated under the following assumptions: (1) Velocities at impeller inlet at radius R_1 are radial; (2) the inflow velocity f at R_1 and the absolute tangential velocity component w at the impeller exit at radius R_2 are given by $f/u = (1-2w/u)\tan\beta$; (3) $w/u = (\tan\beta - F/u)/(2\tan\beta - F/u)$; (4) the point of operation is between inflow zero and maximum hydraulic efficiency (mhe). Here $u = \omega R_2$, F is inflow velocity for zero head, and β the blade exit angle.

The head is then assumed to be $uw/g + kw^2/2g$, where $k = 1-R_1^2/R_2^2$. Head values are calculated for inflow velocities f_{mhe}, $0.5f_{mhe}$ and zero. The agreement between actual and calculated heads is quite satisfactory.

The formulas connecting f, w, and F are derived by considering a flow in a bent pipe in interaction with a straight row of vanes moving across the pipe with velocity u. In the formula for the head the increase of angular momentum accounts for the first term, while the second term represents a centrifugal head. To this reviewer, the derivation of the f, w, and F relations seems unconvincing, and the addition of a centrifugal term in the head formula seems incorrect. G. Kuerti, USA

Experimental Flow Equipment and Technique

(See also Revs. 234, 369)

361. A. v. Baranoff, "Tunnel correction for compressible subsonic flow," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1162*, July 1947, pp. 1–19 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1272, 1940).

The subsonic first order wind-tunnel wall corrections are presented for circular and rectangular tunnels. The effect of the finite body volume is investigated by considering an equivalent doublet and using the linearized equation of compressible potential flow. The effect on the trailing vortices in both a closed and an open-throat circular tunnel are investigated by replacing the lifting surface by an equivalent horseshoe vortex with infinitesimal span. E. V. Laitone, USA

362. Fernand Charron, "Motion of a viscous fluid between concentric spheres (Mouvement d'un fluide visqueux entre sphères concentriques)," *C. R. Acad. Sci., Paris*, Feb. 10, 1947, vol. 224, pp. 373–374.

Experiments made on the viscous flow of mixtures of vaseline and oil between relatively rotating spherical surfaces showed that fluid particles followed a complicated path in the spherical surface. The measured couple on the rotating sphere was not proportional to the angular speed, and deviated more from proportionality as the speed increased. A simplified analytical study is included. R. C. Binder, USA

363. Laszlo Kovasznay, "Calibration and measurement in turbulence research by the hot-wire method," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1130*, June 1947, pp. 1–36 (transl. from publication of Müegyetem Aerodinamikai, Budapest, 1943).

This translation from the Hungarian of Kovasznay's paper appears four years after the original, and to some extent the information contained in it is known from private communications. It includes a comprehensive account of the theory of the hot-wire anemometer, including the effects of circuit constants on the sensitivity and time constant of the hot-wire.

It is shown that the time constant of the hot-wire response is the same whether the response is to a variation of wind velocity or to a variation of heating current. From this the validity of the electrical method of determining the necessary compensation is confirmed, and an extremely simple and accurate method of measuring this compensation is described, in which the response to a square-wave variation in heating current is observed on a cathode-ray oscillograph. The square-wave variation is produced by a mechanically actuated switch rapidly short-circuiting a small series resistance.

The turbulence-measuring equipment developed by the author is then described, using a compensation circuit with a mutual inductance in place of the more usual circuits using capacitances or self-inductances. An accuracy of approximately 4 per cent is claimed in measurements of turbulent velocities, which is probably better than contemporary measurements.

A. A. Townsend, England

364. R. Weller, "The optical investigation of fluid flow," *J. appl. Mech.*, June 1947, vol. 14, pp. 103–107.

The methods of photoelastic stress analysis are applied to the measurement of flow in viscous fluids. Double refraction in certain very viscous fluids is shown to be proportional to the velocity gradient by the "photoviscous law"

$$\text{(observed fringe order)} = \frac{\text{(thickness)}}{\text{(fringe value)}} \times \text{(velocity gradient)}$$

An apparatus to measure the characteristics of various liquids is described. It consists of two coaxial cylinders. The liquid is contained in the outer stationary cylinder, and the inner concentric cylinder is immersed in the liquid and rotated at various speeds. Polarized light passes through the shearing liquid by means of windows. The sensitivity of several high-molecular-weight organic materials in solution is shown by plotting compensator shift as a function of the angular velocity of the inner cylinder. A fluid test tunnel is also described, with several flow patterns produced by this method.

C. O. Dohrenwend, USA

Hydraulics; Transport of Solids; Cavitation

(See also Revs. 202, 314, 322)

365. Jack Allen, "Model experiments in relation to harbors and waterways," *J. Instn. civ. Engrs.*, Oct. 1947, vol. 28, pp. 377–411.

This is a descriptive paper which refers briefly to five previously reported experiments with river and harbor models. Considerable discussion is given of the interesting question of using different scales in the vertical and horizontal directions, so as to obtain the same laminar or turbulent motion in the model as in the prototype, with a practical degree of smoothness of model surfaces. A bibliography and many photographs and plans of models and prototypes are included. Gino Moretti, Italy

366. G. S. Peterson, "Orifice discharge coefficients in the viscous-flow range," *Trans. Amer. Soc. mech. Engrs.*, Oct. 1947, vol. 69, pp. 765–768.

Few published data are available on coefficients for small orifices operating in the viscous-flow range. This paper presents results obtained from calibrating two orifices used in measuring lubricating-oil flow rates in aviation engines during flight.

The orifice disks were made of 1/64-in. flat steel stock with ratios of orifice diameter a to pipe diameter A of 0.40, 0.50, 0.555, and 0.656. The assemblies were mounted in an oil-filter well,

which in turn was fitted in a special test housing representing the aviation engine.

The pressure drop across the orifice was determined by means of a bellows-type differential gage with diaphragm-type pressure transmitter in each pressure line. Flow rates were determined by means of a weighing tank included in the circuit. Tests were run with differential heads h across the orifice ranging between 1 in. and 30 in. of mercury, using SAE 60 oil at temperatures corresponding to a viscosity range between 175 and 3000 SSU.

The coefficient C in the formula $Q = Ca\sqrt{2gh/(1-a^2/A^2)}$ was calculated and plotted against the Reynolds number. Large changes in coefficient values were found for small changes in diameter ratio and for relatively small changes in Reynolds number. The author believes it advisable, therefore, to avoid the use of orifices under the conditions where abrupt changes in coefficient values occur. Karl E. Schoenherr, USA

367. R. C. Brown, "The fundamental concepts concerning surface tension and capillarity," *Proc. phys. Soc. London*, May 1947, vol. 59, pp. 429–448.

In order to clarify the elementary considerations of surface tension, surface energy, and capillary rise, fundamental concepts are discussed and basic equations derived from a consideration of the cohesive forces of matter. In contradiction to the rather widely held belief that surface energy is the true physical quantity while surface tension is merely a useful fiction, surface tension is shown to be a real force directly related to the surface energy. A comparison of the forces on elementary areas perpendicular and parallel to a free surface indicates a difference in pressure intensity in the two directions near the surface, and points to the existence of a tensile force parallel to the surface in this region.

The equation relating the total and free surface energy is presented as obtained from the laws of thermodynamics, and an equivalent relationship is derived which provides a somewhat less abstract conception of the distinction between the two forms of surface energy.

Although the concepts of cohesion and surface energy are equally applicable to liquids and solids, the concept of surface tension is shown to be inapplicable to solids because they have no tendency to contract at a surface and require no external force to maintain equilibrium. The roles played by both the solid and the liquid at an interface are examined in the light of the concepts presented, and the capillary-rise expression is derived from consideration of the negative surface tension at such an interface.

John S. McNown, USA

368. Giulio De Marchi, "Forms of the free surface of permanent streams with progressively increasing or decreasing flow in a constant cross section (Des formes de la surface libre de courtants permanents avec débit progressivement croissant ou progressivement décroissant dans un canal de section constante)," *Rev. gen. Hyd.*, Mar.–Apr., 1947, vol. 13, pp. 81–85.

By assuming that the channel slope neutralizes frictional losses, the differential equation for the free surface in a uniform channel is written

$$\frac{dh}{dx} = A\,\frac{\sigma a Q}{g\sigma^3 - lQ^2}$$

where h is the depth of flow, σ the cross-sectional area, a the rate of change in the flow Q, and l the free surface width. For a decreasing Q the energy relation indicates a value of $+1$ for A, while for increasing Q the momentum relation gives a value of -2.

Integration of the equation for a rectangular channel in which the flow leaves over a side weir of height c, indicates three possible flow conditions depending on the relative magnitudes of h, c, and the value of the specific energy at the upstream or downstream end e_o. In two conditions the flow is tranquil and the depth increases, while in the third the flow is rapid and the depth decreases. Laboratory tests are mentioned which checked the theory for the tranquil flow region with $c > 0.75\ e_o$. Integration of the equation for a rectangular channel with increasing flow (flow into a gutter) yields two cases, one with rapid flow in which the depth increases in the flow direction, and one with tranquil flow in which the depth decreases.

Since more general analyses of the last case [T. R. Camp, *Trans. Amer. Soc. civ. Engrs.*, 1940, vol. 105, p. 606] have indicated that the effects of friction and channel slope do not neutralize each other as assumed by the author, and hence give different solutions depending upon the friction and slope, it is probable that these effects are of importance in the other cases also.

J. M. Robertson, USA

369. J. W. Bremner, "Calculation of the error due to leakage in the hydraulic ram method of pressure calibration," *Phil. Mag. London*, Aug. 1947, vol. 38, pp. 564–573.

A common method of calibrating a pressure gage is to connect it to a cylinder filled with oil acted on by a loaded ram, the pressure being assumed equal to the ram load divided by cross-sectional area. An analytical study is presented of the error arising from the movement of the ram in the cylinder and the flow of liquid between them. It is shown that such an error occurs, but is not likely to exceed one per cent. Experimental data are presented which confirm the analysis. The error for a particular instrument can be determined from timing the fall of the ram, and the viscosity of the oil. R. C. Binder, USA

370. L. F. Moody, "An approximate formula for pipe friction factors," *Mech. Engng.*, Dec. 1947, vol. 69, pp. 1005–1006.

This addendum to a previous paper by the author on friction factors for pipe flow presents an approximation to the Colebrook equation which permits direct solution for the pipe friction factor in terms of the surface roughness and the Reynolds number. It is stated that the formula is applicable over the usual range of engineering problems and agrees with the Colebrook equation within an error of ± 5 per cent. George Gerard, USA.

Marine Propulsion

371. F. W. S. Locke, Jr., "A preliminary correlation of the behavior of water rudders on seaplanes and flying boats," *Nat. Advis. Comm. for Aero. Tech. Note No. 1387*, Aug. 1947, pp. 1–8.

In want of any systematic research upon water rudders for seaplanes and flying boats, this short note presents some empirical data for the help of the designer. Gino Moretti, Italy

372. L. C. Burrill, "On propeller theory," *Trans. Instn. Engrs. Shipb. Scot.*, 1946–1947, vol. 90, pp. 449–477.

The "axial-momentum" and "blade-element" theories of ship propellers are reviewed, with brief derivations of the equations for thrust, torque, and efficiency. Corrections for rotation of the slipstream are added. The two theories predict different axial velocities at the propeller and different maximum efficiencies. An assumption that free vortices leave the trailing edge of each blade improves the blade-element theory, providing both the axial inflow given by the momentum theory and an induced angular velocity of the water. Stated differently, the vortex theory provides an essential aspect-ratio correction for lift and drag forces on blade elements.

This reconciles the two older theories for the case of propellers

with many blades. Corrections for a small number of blades are outlined, with references to the literature. Equations are presented to predict thrust, torque, and efficiency, and in principle to allow designing for maximum efficiency.

Thirty references (from 1865–1944) and a discussion of this article are appended. A. O. Williams, Jr., USA

Lubrication; Wear

373. Fernand Charron, "Viscosity under rapidly changing pressure (Viscosité sous pression rapidement variable)," *C. R. Acad. Sci., Paris.*, Nov. 17, 1947, vol. 225, pp. 919–921.

The author has investigated the question whether the change in viscosity with pressure of lubricants is an instantaneous change, or whether the viscosity change is a function of the time. While the problem is complicated by temperature changes due to the pressure changes, his experiments with a "ballistic viscosimeter" [*C. R. Acad. Sci., Paris*, 1947, vol. 224, pp. 1472–1474] have indicated that the viscosity does not reach its steady-state value instantaneously, but that there is a certain hysteresis or time lag in the viscosity increase.

Further experiments are planned to prove this point and to investigate it further. The effect is important for those applications of lubricants where shock or impact in the bearings may cause very high pressures of short time duration.

Erle I. Shobert, II, USA

374. K. G. Brummage, "An electron-diffraction study of the heating of straight-chain organic films and its application to lubrication," *Proc. roy. Soc., London, Ser. A*, Nov. 11, 1947, vol. 191, pp. 243–252.

Thin films of normal paraffins, fatty acids, and esters upon surfaces of aluminum, cadmium, copper, nickel, silver, mild steel and stainless steel, were examined by electron diffraction during heating. In all cases the diffraction patterns showed the films to change from an orderly crystalline arrangement of molecules to an expanded state of individual molecules at a rather definite temperature. This disorientation was found to be reversible, the orderly arrangement returning when the specimen was cooled.

Films deposited by either the Langmuir-Blodgett technique, or from isohexane, gave the same disorientation temperatures within an experimental precision of ±2 to ±5 C. The disorientation temperature was found to increase with film thickness and to depend also upon the substrate and film materials. While no explanation of the influence of these variables upon the disorientation temperature is offered, a correlation was observed between the bulk melting point of the film material and the disorientation temperature for members of a homologous series.

The disorientation temperature was found to correspond to the temperature at which the smooth friction of a lightly loaded, slowly moving slider abruptly increased with increase in temperature. The temperature marking the onset of "stick-slip" sliding was generally found to lie above the disorientation temperature, which indicates that even an expanded film is adsorbed on the metal surface. It is suggested that the few cases in which the smooth friction transition point is observed to lie above the disorientation temperature are due to the formation of a chemical compound between the substrate and the film.

M. C. Shaw, USA

375. J. M. Stone and A. F. Underwood, "Load-carrying capacity of journal bearings," *Soc. auto. Eng. Quart. Trans.*, Jan. 1947, vol. 1, no. 1, pp. 56–57.

A derivation is given of a form of Reynold's equation for the pressure distribution in the oil film of a journal bearing, for bearings in which there is rotation not only of the journal but also of the direction of the load. It indicates that the load-carrying capacity of such a bearing (defined as the magnitude of the load required to produce a given minimum oil-film thickness) is proportional to the absolute value of the quantity 2 × (load direction rpm) − (journal rpm).

This leads to the conclusions that: (1) a journal bearing may have a finite load-carrying capacity when there is no relative motion between journal and bearing, and (2) there is a condition for which a journal bearing will have zero load-carrying capacity, even though there is relative surface velocity of journal and bearing. Apparently, contrary conclusions reached on the basis of unidirectional load tests are due to their being particular cases of the general case studied here.

A test setup is described with which minimum oil-film thickness and an approximation to the friction torque on the bearing can be measured. Results of tests on various bearing types are given to substantiate the above analysis and conclusions.

Frank J. Maginnis, USA

Dynamics of Meteorology and Oceanography

(See also Rev. 365)

376. H. U. Sverdrup, "Wind-driven currents in a baroclinic ocean; with application to the equatorial currents of the Eastern Pacific," *Proc. nat. Acad. Sci., Wash.*, Nov. 1947, vol. 33, pp. 318–326.

The author investigates the relationships between the wind-driven ocean currents near the equator, under the assumption that the horizontal velocities and pressure gradients vanish at a moderate depth (of the order of 1000 meters), that the relative acceleration may be neglected in comparison with the Coriolis acceleration due to the earth's rotation, and that lateral stresses are not significant. The equations of motion are approximately integrated, and the currents to the west of a meridional boundary are computed in terms of the surface wind stress distributions.

The observed wind distributions are then used to estimate the currents for the mid-Pacific region, and these results are compared with the results obtained from more direct oceanographic observations. A very good agreement in the latitudinal current distribution is obtained; in particular, the necessity for the existence of the reverse equatorial current which is embedded in the trade-wind currents is demonstrated. H. J. Stewart, USA

377. B. Haurwitz, "Comments on the sea-breeze circulation," *J. Meteorol.*, Feb. 1947, vol. 4, pp. 1–8.

It is shown: (*a*) that the effect of friction is to reduce the phase lag of maximum sea breeze after maximum land-sea temperature difference; (*b*) that neglect of the acceleration in the equations of sea-breeze motion leads to a result inconsistent with observations; and (*c*) that consideration of the effects of friction (assumed proportional to speed) and earth's rotation, lead to a diurnal variation of wind, corresponding to a given sinusoidal pressure oscillation, which agrees fairly well with observed variation at Boston. In the last development the treatment is similar to that by E. Gold [*Phil. Mag.*, 1910, vol. 19, p. 26] but no reference is made to Gold's work. M. Neiburger, USA

378. H. A. Panofsky, "Radiative cooling in the lower layers of an atmosphere warmer than the ground," *J. Meteorol.*, Feb. 1947, vol. 4, pp. 35–37.

The question of the relative importance of radiation as a mechanism of heat transfer in the atmosphere is here discussed in one of its many aspects, namely, radiative cooling of warm air flowing

over a surface 10 C colder. Although somewhat extreme assumptions are made and not completely clarified, the results so obtained are interesting in their orders of magnitude. The methods of the Elsasser radiation chart are used to calculate the rate of cooling in air containing both water vapor and carbon dioxide. This rate is found to be much smaller than that actually observed for wind speeds greater than 15 mph. Eddy turbulence is suggested to be the important mechanism of heat transfer in these cases. Joanne G. Starr, USA

379. B. Gutenberg, "Microseisms and weather forecasting," *J. Meteorol.*, Feb. 1947, vol. 4, pp. 21–28.

Microseisms are surface waves (their energy is propagated principally in a layer about 20 km thick, according to the author) which can be detected by sensitive seismographs. They can be propagated to great distances if they do not encounter geologic discontinuities. The point of origin can be determined by measuring the differences in time of arrival at three stations at the corners of a triangle with sides one or two miles long.

The author studies particularly microseisms which are connected with atmospheric disturbances. He considers various explanations for their formation, and thinks that their energy is derived from that of surges raised up by tempests on the ocean; however he is unable to show how the energy is transmitted.

He then describes the method used by the U. S. Navy Department for locating hurricanes and following their course with the aid of microseisms. L. J. Tison, Belgium

380. F. H. Schmidt, "An elementary theory of the land- and sea-breeze circulation," *J. Meteorol.*, Feb. 1947, vol. 4, pp. 9–15.

An analytic expression approximating the temperature variation in the vicinity of a straight coast line is assumed. From it an expression for the density variation is derived, using additional approximations, and introduced into the equations of motion for both equatorial latitudes (Coriolis force neglected) and temperature latitudes, by way of an equation for the pressure variation at any level. The latter involves an approximation of questionable validity. Friction is assumed proportional to speed.

In the equatorial case the solution indicates that the sea breeze is delayed after the temperature oscillation by an amount which is smaller the larger the coefficient of friction, and the sea-breeze amplitude is proportional to the amplitude of the density variation at the ground. Adjustment of the parameters permits the theoretical curves for variation with height of the starting time and amplitude of sea breeze to fit the observed variation at Batavia.

For the temperature latitude the solution gives a velocity component along the coast out of phase with the cross-coast component, causing the sea breeze to veer with time, in addition to the lag between it and the temperature variation. Comparison of this theory with the variation of surface wind direction during one day in Holland shows considerable similarity.

M. Neiburger, USA

Ballistics

(See also Rev. 210)

381. R. Hill and D. C. Pack, "An investigation, by the method of characteristics, of the lateral expansion of the gases behind a detonating slab of explosives," *Proc. roy. Soc., London, Ser. A*, Dec. 3, 1947, vol. 191, pp. 524–541.

An uncased explosive charge is detonated in a fluid. Considering the block of explosive and the gas and the fluid fields to be two-dimensional, the hydrodynamical equations are integrated, using the method of characteristics. The equations are converted into difference relations and then integrated numerically. The details of this process are given.

As to the boundary conditions, a serious difficulty arises near to and on the block face, since the gradients of the characteristics tend to infinity as the face is approached. The method adopted is to obtain analytical solutions in series form, which are valid near the face and close to the corners, thus substituting, for the boundary conditions at the face, boundary conditions some distance out into the field where the gradients are sufficiently small to permit the use of the equations of characteristics.

Three different types of expansions for the velocity potential are assumed. A more general method of approach demonstrates that a simple Meyer expansion provides a remarkably accurate solution over a large part of the gas field. Diagrams of the results obtained show the distributions of the velocity components and the pressure at given distances from the block face.

The explosive is taken to be TNT with a velocity of detonation of 6790 meters per sec and a local velocity of sound just behind the detonation wave of 4697 meters per sec which gives, according to the Chapman-Jouquet condition, a gas velocity of 2093 meters per sec. The shock wave is found to be straight for a distance from the corner of nearly 6 times the thickness of the charge. Calculated pressures are: behind the shock wave 306.4 atm and at the mid-point of the face 16×10^{10} dynes per cm^2.

W. Weibull, Sweden

382. G. Stamm, "Investigations of pressure distribution on fast flying bodies," *Nat. Advis. Comm. for Aero. Tech. Memo. No. 1101*, Sept. 1946, pp. 1–18 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 8103).

This paper discusses measured static pressure change on and behind the bow wave of two supersonic projectiles. A pressure rise of 0.5 atm occurred on the bow wave of one projectile at a distance of 3 meters from the projectile, while tests had shown that an explosive charge of 20 grams at a distance of 3 meters or a static pressure change of 0.5 atm is destructive to windows. The destructiveness of this bow wave is thus correlated with that of an explosive charge. Henry J. Barten, USA

Thermodynamics

(See also Revs. 331, 332, 355)

383. B. Davydov, "Quantum mechanics and thermodynamic irreversibility" (in English), *J. Phys. Acad. Sci. USSR*, 1947, vol. 11, no. 1, pp. 33–43.

The problem of thermodynamic irreversibility as expressed by the principle of the increase of entropy is examined in this paper from the viewpoint of quantum mechanics. It is observed that classical statistical mechanics leads to an essentially reversible status of physical events. On the other hand the irreversibility of measurements in quantum mechanics leads to a permanent increase in entropy.

It is possible furthermore to associate thermodynamic entropy with a "quantum entropy" defined by $\sigma = \rho - Spur\ \rho \ln \rho$, where ρ is the density matrix. It is pointed out that while it is not proper to equate these two entropies in general, certain relations exist which allow the conclusions of thermodynamics to correlate positively with those of quantum mechanics. N. A. Hall, USA

384. J. H. Childs, R. J. McCafferty, and O. W. Surine, "Effect of combustor-inlet conditions on combustion in turbojet engines," *Soc. auto. Engrs. Quart. Trans.*, Apr. 1947, vol. 1, pp. 266–278.

The authors describe tests of an early type of annular combus-

tion chamber for the Westinghouse 19B turbojet engine, performed at the NACA Cleveland Laboratory. Measurements were made of the relation between temperature rise across the chamber and fuel-air ratio, for differing inlet air pressures, temperatures and velocities.

At any given low pressure, temperature, or high velocity, the temperature rise versus fuel-air ratio curves showed a maximum temperature rise at a fuel-air ratio much lower than stoichiometric value. The combustion efficiency and maximum temperature rise obtainable decreased, and resonant combustion increased if (*a*) the inlet pressure or (*b*) the inlet temperature were decreased, and if (*c*) the inlet velocity was increased. AN-F-22 fuel was compared with AN-F-28. The results were in good agreement with those from altitude tests of the engine, and indicate that the limits to altitude operation of the engine were set by the combustion chamber. W. R. Hawthorne, USA

385. Max Serruys, "Influence of the nature of the fuel mixture on the velocity of gases through a nozzle (Influence de la nature du mélange combustible sur la vitesse des gaz éjectés par une tuyère)," *C. R. Acad. Sci., Paris*, Jan. 20, 1947, vol. 224, pp. 190–191.

The author shows that, assuming k/γ approximately constant (where k is the exponent for a polytropic change, γ the ratio of the specific heats), any increase in mean molecular specific heat of gases tends to decrease k, and therefore also γ. He shows that, in general, the escape velocity from the nozzle will decrease when the mean molecular specific heat of gases decreases.

A. D. Kafadar, USA

Heat Transfer

(See also Revs. 203, 332, 378)

386. A. Scriabin, "Heat transfer and hydraulic losses in needle-type gas heaters" (in Russian), *Bull. Acad. Sci. URSS (Ser. techn. Sci.)*, 1947, no. 2, pp. 189–203.

This is an experimental study of heat transfer and flow resistance for air flowing over heated cast-iron plates covered with rows of needlelike projections normal to the surface. Thirty series of tests are reported, with various combinations of shape, spacing, and arrangement of projections.

The results are interpreted by the principles of dimensional analysis or thermodynamic similitude, using the common dimensionless numbers Re, Nu, Pé, Pr, Eu. Two combinations of numbers plotted against Re are used in studying the optimum shape characteristics. However, the author observes that the choice of a shape-characteristic criterion needs much further study.

The results of this investigation can be suitably represented by the relation $\mathrm{Nu} = A(\mathrm{Re})^n$ where A varies between 0.41 and 3.0, n between 0.44 and 0.68 for values of Re between 2×10^3 and 40×10^3 (it is to be noted that this relation is similar to that obtained by other investigators for pipe flow, radiators, finned surfaces, etc.).

The over-all transmission coefficient varies but little with the needle shape. The surface ratio has more effect, best results being given by the ratio 3.2 for total surface to plane surface, and the ratio of 0.19 for the ratio of needle cross section to plane surface. D. Jacovleff, Belgium

387. C. F. Kayan, "Effect of floor slab on building structure temperatures and heat flow," *Heat. Pip. Air Condit.*, June 1947, vol. 19, pp. 103–111.

The study of the steady-state flow of heat in complex structures is simple in principle but complicated in execution. Conventional mathematical technique first replaces the continuous structure by a mesh (essentially replacing the partial differential equation governing heat flow by difference equations), and then uses successive approximations (relaxation methods) to find the lines of heat flow and the isothermals. A far simpler procedure is to exploit the analogy between heat flow and the flow of electricity by setting up an electrical model equivalent to the thermal problem under study.

In this paper the analog technique is used to study how the heat-flow pattern through a wall is affected by a floor or ceiling. Schematically one can think of the floor as an extended horizontal surface fin attached to a long vertical slab. This effectively changes a simple one-dimensional problem into one involving two-dimensional flow. The author studies the problem under various assumptions concerning the value of outside conductance (which is related to such things as wind velocity). Schematic isothermal plots and temperature distributions at the surfaces of the wall and slab are given. Such information is of current interest in connection with radiant panel heating. A more complicated example involving both structural and insulating materials in a cold-storage problem is also treated. Benjamin Epstein, USA

388. Hunter Rouse, "Gravitational diffusion from a boundary source in two-dimensional flow," *J. appl. Mech.*, Sept. 1947, vol. 14, no. 3, pp. 225–228.

This paper deals with convection currents of heated air from a line of burners. The mathematical analysis makes use of gravitational diffusion from a boundary source. The analysis is limited to two-dimensional flow and results in the following conclusions: (1) The characteristics of the diffusion pattern depend solely upon the Froude number of the flow; (2) The velocity characteristic is constant from section to section and the diffusion coefficient is a linear function of distance from the source.

An essential distinction between diffusion due to initial turbulence of the ambient fluid and that due to gravitational convection is that turbulence is subject to gradual decay due to viscous action, while the convection process continuously increases with distance downstream.

Test data shown permits the verification of the proposed theory as well as the numerical evaluation of the constant. Although there is some deviation of the test points from the mathematical curve, the measured values provide a sound verification of the theory. The proposed theory, particularly if it were expanded to the three-dimensional case, is important as it makes possible the analysis of the "fido" fog disposal systems and has other technical applications as well. H. E. Sheets, USA

389. Stuart R. Brinkley, Jr., "Heat transfer between a fluid and a porous solid generating heat," *J. appl. Phys.*, June 1947, vol. 18, pp. 582–585.

The analysis of Anzelius and Schumann for the transfer of heat between a porous body and a fluid flowing through it is extended to include the case where the porous material generates heat.

Expressions are obtained for the temperature of the solid and fluid as a function of time and position, based on the following idealizations: (1) Thermal conduction in solid or fluid is negligible compared with the transfer of heat from fluid to solid. (2) Thermal diffusivity of solid is infinite. (3) Specific heat and densities of solid and fluid are constant. (4) Temperature of solid and fluid are functions only of distance measured in the direction of flow, and time. (5) The heat source function is a linear function of the temperature of the solid with coefficients independent of position and time.

Applications to systems such as catalytic reactors are to be described in later papers. R. C. Martinelli, USA